Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "0 Independent test suites"

Test results for the 175 problems in "Apostol Problems.m"

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt$$

Optimal (type 4, 172 leaves, 2 steps):

$$\frac{2}{5}t\sqrt{4+t^3}-\frac{8\times 2^{2/3}\sqrt{2+\sqrt{3}}\,\left(2^{2/3}+t\right)\sqrt{\frac{2\,2^{1/3}-2^{2/3}t+t^2}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}\left(1-\sqrt{3}\right)+t}{2^{2/3}\left(1+\sqrt{3}\right)+t}\right],-7-4\sqrt{3}\right]}{5\times 3^{1/4}\sqrt{\frac{2^{2/3}+t}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\ \sqrt{4+t^3}}$$

Result (type 4, 122 leaves):

$$\frac{1}{15\sqrt{4+t^3}}\left(6t\left(4+t^3\right)-8\left(-2\right)^{1/6}3^{3/4}\sqrt{-\left(-1\right)^{1/6}\left(2\left(-1\right)^{2/3}+2^{1/3}t\right)}\ \sqrt{4+2\left(-2\right)^{1/3}t+\left(-2\right)^{2/3}t^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(2+2^{1/3}t\right)}}{2\times 3^{1/4}}\right],\left(-1\right)^{1/3}\right]\right)$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int x^4\left(1+x^5\right)^5\,dx$$

Optimal (type 1, 11 leaves, 1 step):

$$\frac{1}{30}\left(1+x^5\right)^6$$

Result (type 1, 43 leaves):

$$\frac{x^5}{5}+\frac{x^{10}}{2}+\frac{2\,x^{15}}{3}+\frac{x^{20}}{2}+\frac{x^{25}}{5}+\frac{x^{30}}{30}$$

Problem 51: Result more than twice size of optimal antiderivative.

$$\int (1-x)^{20}\,x^4\,dx$$

Optimal (type 1, 56 leaves, 2 steps):

$$-\frac{1}{21}(1-x)^{21}+\frac{2}{11}(1-x)^{22}-\frac{6}{23}(1-x)^{23}+\frac{1}{6}(1-x)^{24}-\frac{1}{25}(1-x)^{25}$$

Result (type 1, 140 leaves):

$$\frac{x^5}{5}-\frac{10\,x^6}{3}+\frac{190\,x^7}{7}-\frac{285\,x^8}{2}+\frac{1615\,x^9}{3}-\frac{7752\,x^{10}}{5}+\frac{38\,760\,x^{11}}{11}-6460\,x^{12}+9690\,x^{13}-\frac{83\,980\,x^{14}}{7}+$$
$$\frac{184\,756\,x^{15}}{15}-\frac{20\,995\,x^{16}}{2}+7410\,x^{17}-\frac{12\,920\,x^{18}}{3}+2040\,x^{19}-\frac{3876\,x^{20}}{5}+\frac{1615\,x^{21}}{7}-\frac{570\,x^{22}}{11}+\frac{190\,x^{23}}{23}-\frac{5\,x^{24}}{6}+\frac{x^{25}}{25}$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1+3\,\mathrm{Cos}[x]^2}\,\mathrm{Sin}[2\,x]\,dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$-\frac{2}{9}\left(4-3\,\mathrm{Sin}[x]^2\right)^{3/2}$$

Result (type 3, 49 leaves):

$$\frac{5\sqrt{5}-5\sqrt{5+3\,\mathrm{Cos}[2\,x]}-3\,\mathrm{Cos}[2\,x]\sqrt{5+3\,\mathrm{Cos}[2\,x]}}{9\sqrt{2}}$$

Problem 83: Result more than twice size of optimal antiderivative.

$$\int \mathrm{ArcSec}[x]\,dx$$

Optimal (type 3, 19 leaves, 4 steps):

$$x\,\text{ArcSec}[x]-\text{ArcTanh}\left[\sqrt{1-\frac{1}{x^2}}\right]$$

Result (type 3, 64 leaves):

$$x\,\text{ArcSec}[x]-\frac{\sqrt{-1+x^2}\left(-\text{Log}\left[1-\frac{x}{\sqrt{-1+x^2}}\right]+\text{Log}\left[1+\frac{x}{\sqrt{-1+x^2}}\right]\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

Problem 84: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCsc}[x]\,dx$$

Optimal (type 3, 17 leaves, 4 steps):

$$x\,\text{ArcCsc}[x]+\text{ArcTanh}\left[\sqrt{1-\frac{1}{x^2}}\right]$$

Result (type 3, 64 leaves):

$$x\,\text{ArcCsc}[x]+\frac{\sqrt{-1+x^2}\left(-\text{Log}\left[1-\frac{x}{\sqrt{-1+x^2}}\right]+\text{Log}\left[1+\frac{x}{\sqrt{-1+x^2}}\right]\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\text{Cos}[x]+\text{Sin}[x]}\,dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]-\text{Sin}[x]}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 24 leaves):

$$(-1-i)\,(-1)^{3/4}\,\text{ArcTanh}\left[\frac{-1+\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right]$$

Problem 154: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x + x^2}}\, dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$2\,\text{ArcTanh}\left[\frac{x}{\sqrt{x + x^2}}\right]$$

Result (type 3, 29 leaves):

$$\frac{2\sqrt{x}\,\sqrt{1+x}\,\text{ArcSinh}\left[\sqrt{x}\right]}{\sqrt{x\,(1+x)}}$$

Problem 175: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1 + t^3}}\, dt$$

Optimal (type 4, 103 leaves, 1 step):

$$\frac{2\sqrt{2+\sqrt{3}}\,(1+t)\sqrt{\frac{1-t+t^2}{\left(1+\sqrt{3}+t\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right], -7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+t}{\left(1+\sqrt{3}+t\right)^2}}\,\sqrt{1+t^3}}$$

Result (type 4, 88 leaves):

$$\frac{1}{3^{1/4}\sqrt{1+t^3}}2\,(-1)^{1/6}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+t\right)}\,\sqrt{1+(-1)^{1/3}t+(-1)^{2/3}t^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+t)}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Test results for the 35 problems in "Bondarenko Problems.m"

Problem 1: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2} + \text{Cos}[z] + \text{Sin}[z]}\, dz$$

Optimal (type 3, 22 leaves, 1 step):

$$-\frac{1-\sqrt{2}\,\text{Sin}[z]}{\text{Cos}[z]-\text{Sin}[z]}$$

Result (type 3, 77 leaves):

$$\frac{-\left(\left(1+3\,i\right)+\sqrt{2}\right)\text{Cos}\left[\frac{z}{2}\right]+\left(\left(1+i\right)-i\,\sqrt{2}\right)\text{Sin}\left[\frac{z}{2}\right]}{\left(\left(1+i\right)+\sqrt{2}\right)\text{Cos}\left[\frac{z}{2}\right]+i\left(\left(-1-i\right)+\sqrt{2}\right)\text{Sin}\left[\frac{z}{2}\right]}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Log}[1+x]}{x\sqrt{1+\sqrt{1+x}}}\,dx$$

Optimal (type 4, 291 leaves, ? steps):

$$-8\,\text{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right]-\frac{2\,\text{Log}[1+x]}{\sqrt{1+\sqrt{1+x}}}-\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right]\text{Log}[1+x]+$$
$$2\sqrt{2}\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1-\sqrt{1+\sqrt{1+x}}\right]-2\sqrt{2}\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]+\sqrt{2}\,\text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]-$$
$$\sqrt{2}\,\text{PolyLog}\left[2,\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]-\sqrt{2}\,\text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]+\sqrt{2}\,\text{PolyLog}\left[2,\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]$$

Result (type 4, 816 leaves):

$$-\frac{4\left(2+\text{Log}\left[1+\sqrt{1+x}\right]\right)}{\sqrt{1+\sqrt{1+x}}}-4\left(-1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right)\left(-1+\text{Log}\left[-1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]\right)-4\left(1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right)\left(-1+\text{Log}\left[1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]\right)+\sqrt{2}$$

$$\left(\text{Log}[1+x]-2\left(\text{Log}\left[1+\sqrt{1+x}\right]+\text{Log}\left[-1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]+\text{Log}\left[1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]\right)\right)\left(\text{Log}\left[\sqrt{2}-\frac{2}{\sqrt{1+\sqrt{1+x}}}\right]-\text{Log}\left[\sqrt{2}+\frac{2}{\sqrt{1+\sqrt{1+x}}}\right]\right)-$$

$$\frac{1}{2\sqrt{1+\sqrt{1+x}}}\left(\text{Log}[1+x]-2\left(\text{Log}\left[1+\sqrt{1+x}\right]+\text{Log}\left[-1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]+\text{Log}\left[1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]\right)\right)$$

$$\left(4+\sqrt{2}\sqrt{1+\sqrt{1+x}}\,\text{Log}\left[\sqrt{2}-\frac{2}{\sqrt{1+\sqrt{1+x}}}\right]-\sqrt{2}\sqrt{1+\sqrt{1+x}}\,\text{Log}\left[\sqrt{2}+\frac{2}{\sqrt{1+\sqrt{1+x}}}\right]\right)+$$

$$\sqrt{2}\left(-\text{Log}\left[1+\sqrt{1+x}\right]\text{Log}\left[1+\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+x}}}\right]+2\,\text{PolyLog}\left[2,-\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+x}}}\right]\right)+$$

$$\sqrt{2}\left(\text{Log}\left[1+\sqrt{1+x}\right]\text{Log}\left[1-\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+x}}}\right]-2\,\text{PolyLog}\left[2,\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+x}}}\right]\right)-$$

$$\sqrt{2}\left(\text{Log}\left[-1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]\text{Log}\left[\frac{\sqrt{2}+\frac{2}{\sqrt{1+\sqrt{1+x}}}}{2+\sqrt{2}}\right]+\text{PolyLog}\left[2,\frac{2-\frac{2}{\sqrt{1+\sqrt{1+x}}}}{2+\sqrt{2}}\right]\right)+$$

$$\sqrt{2}\left(\text{Log}\left[-1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]\text{Log}\left[1+\frac{2-\frac{2}{\sqrt{1+\sqrt{1+x}}}}{-2+\sqrt{2}}\right]+\text{PolyLog}\left[2,\frac{2\left(-1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right)}{-2+\sqrt{2}}\right]\right)-$$

$$\sqrt{2}\left(\text{Log}\left[1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]\text{Log}\left[\frac{\sqrt{2}+\frac{2}{\sqrt{1+\sqrt{1+x}}}}{-2+\sqrt{2}}\right]+\text{PolyLog}\left[2,-\frac{2\left(1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right)}{-2+\sqrt{2}}\right]\right)+$$

$$\sqrt{2}\left(\text{Log}\left[1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right]\text{Log}\left[1-\frac{2\left(1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right)}{2+\sqrt{2}}\right]+\text{PolyLog}\left[2,\frac{2\left(1+\frac{1}{\sqrt{1+\sqrt{1+x}}}\right)}{2+\sqrt{2}}\right]\right)$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+\sqrt{1+x}}\ \text{Log}[1+x]}{x}\,dx$$

Optimal (type 4, 308 leaves, ? steps):

$$-16\sqrt{1+\sqrt{1+x}}+16\,\text{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right]+4\sqrt{1+\sqrt{1+x}}\ \text{Log}[1+x]-2\sqrt{2}\ \text{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right]\text{Log}[1+x]+4\sqrt{2}\ \text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1-\sqrt{1+\sqrt{1+x}}\right]-4\sqrt{2}\ \text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]+2\sqrt{2}\ \text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]-2\sqrt{2}\ \text{PolyLog}\left[2,\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]-2\sqrt{2}\ \text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]+2\sqrt{2}\ \text{PolyLog}\left[2,\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]$$

Result (type 4, 654 leaves):

$$-16\sqrt{1+\sqrt{1+x}}+4\sqrt{1+\sqrt{1+x}}\ \text{Log}[1+x]+\sqrt{2}\ \text{Log}[1+x]\ \text{Log}\left[\sqrt{2}-\sqrt{1+\sqrt{1+x}}\right]-8\,\text{Log}\left[-1+\sqrt{1+\sqrt{1+x}}\right]-2\sqrt{2}\ \text{Log}\left[\sqrt{2}-\sqrt{1+\sqrt{1+x}}\right]\text{Log}\left[-1+\sqrt{1+\sqrt{1+x}}\right]+8\,\text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]-2\sqrt{2}\ \text{Log}\left[\sqrt{2}-\sqrt{1+\sqrt{1+x}}\right]\text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]-\sqrt{2}\ \text{Log}[1+x]\ \text{Log}\left[\sqrt{2}+\sqrt{1+\sqrt{1+x}}\right]+2\sqrt{2}\ \text{Log}\left[-1+\sqrt{1+\sqrt{1+x}}\right]\text{Log}\left[\sqrt{2}+\sqrt{1+\sqrt{1+x}}\right]+2\sqrt{2}\ \text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]\text{Log}\left[\sqrt{2}+\sqrt{1+\sqrt{1+x}}\right]-2\sqrt{2}\ \text{Log}\left[-1+\sqrt{1+\sqrt{1+x}}\right]\text{Log}\left[\left(-1+\sqrt{2}\right)\left(\sqrt{2}+\sqrt{1+\sqrt{1+x}}\right)\right]-2\sqrt{2}\ \text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]\text{Log}\left[2+\sqrt{2}+\sqrt{1+\sqrt{1+x}}+\sqrt{2}\sqrt{1+\sqrt{1+x}}\right]+2\sqrt{2}\ \text{Log}\left[-1+\sqrt{1+\sqrt{1+x}}\right]\text{Log}\left[1-\left(1+\sqrt{2}\right)\left(-1+\sqrt{1+\sqrt{1+x}}\right)\right]+2\sqrt{2}\ \text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]\text{Log}\left[1-\left(-1+\sqrt{2}\right)\left(1+\sqrt{1+\sqrt{1+x}}\right)\right]-2\sqrt{2}\ \text{PolyLog}\left[2,-\left(-1+\sqrt{2}\right)\left(-1+\sqrt{1+\sqrt{1+x}}\right)\right]+2\sqrt{2}\ \text{PolyLog}\left[2,\left(1+\sqrt{2}\right)\left(-1+\sqrt{1+\sqrt{1+x}}\right)\right]+2\sqrt{2}\ \text{PolyLog}\left[2,\left(-1+\sqrt{2}\right)\left(1+\sqrt{1+\sqrt{1+x}}\right)\right]-2\sqrt{2}\ \text{PolyLog}\left[2,-\left(1+\sqrt{2}\right)\left(1+\sqrt{1+\sqrt{1+x}}\right)\right]$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$-\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\sqrt{x+\sqrt{1+x^2}}+\frac{1}{2}\text{Log}\left[x+\sqrt{1+x^2}\right]-2\,\text{Log}\left[1+\sqrt{x+\sqrt{1+x^2}}\right]$$

Result (type 3, 347 leaves):

$$\frac{1}{12}\left(6x-6\sqrt{1+x^2}+4\left(-2x+\sqrt{1+x^2}\right)\sqrt{x+\sqrt{1+x^2}}-12\,\text{Log}[x]+6\,\text{Log}\left[1+\sqrt{1+x^2}\right]+\frac{1}{1+x^2+x\sqrt{1+x^2}}\right.$$
$$6\sqrt{1+x^2}\left(x+\sqrt{1+x^2}\right)\left(2\sqrt{x+\sqrt{1+x^2}}-2\,\text{ArcTan}\left[\sqrt{x+\sqrt{1+x^2}}\right]+\text{Log}\left[1-\sqrt{x+\sqrt{1+x^2}}\right]-\text{Log}\left[1+\sqrt{x+\sqrt{1+x^2}}\right]\right)+$$
$$\frac{1}{\left(1+x^2+x\sqrt{1+x^2}\right)^2}2\left(1+x^2\right)\left(x+\sqrt{1+x^2}\right)^{3/2}\left(4+2x^2+2x\sqrt{1+x^2}+6\sqrt{x+\sqrt{1+x^2}}\,\text{ArcTan}\left[\sqrt{x+\sqrt{1+x^2}}\right]+\right.$$
$$\left.\left.3\sqrt{x+\sqrt{1+x^2}}\,\text{Log}\left[1-\sqrt{x+\sqrt{1+x^2}}\right]-3\sqrt{x+\sqrt{1+x^2}}\,\text{Log}\left[1+\sqrt{x+\sqrt{1+x^2}}\right]\right)\right)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{1+x}}{x+\sqrt{1+\sqrt{1+x}}}\,dx$$

Optimal (type 3, 41 leaves, 6 steps):

$$2\sqrt{1+x}+\frac{8\,\text{ArcTanh}\left[\frac{1+2\sqrt{1+\sqrt{1+x}}}{\sqrt{5}}\right]}{\sqrt{5}}$$

Result (type 3, 147 leaves):

$$\frac{1}{5}\left(10\sqrt{1+x}-\left(-5+\sqrt{5}\right)\sqrt{2\left(3+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{3-\sqrt{5}}}\sqrt{1+\sqrt{1+x}}\right]+\right.$$
$$\left.2\sqrt{\frac{2}{3+\sqrt{5}}}\left(5+\sqrt{5}\right)\text{ArcTanh}\left[\sqrt{\frac{2}{3+\sqrt{5}}}\sqrt{1+\sqrt{1+x}}\right]-4\sqrt{5}\,\text{ArcTanh}\left[\frac{-1+2\sqrt{1+x}}{\sqrt{5}}\right]\right)$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{x+\sqrt{1-\sqrt{1+x}}}\,dx$$

Optimal (type 3, 73 leaves, 6 steps):

$$2\sqrt{1+x}-4\sqrt{1-\sqrt{1+x}}+\left(1-\sqrt{1+x}\right)^2+\frac{8\,\text{ArcTanh}\left[\frac{1+2\sqrt{1-\sqrt{1+x}}}{\sqrt{5}}\right]}{\sqrt{5}}$$

Result (type 3, 151 leaves):

$$x-4\sqrt{1-\sqrt{1+x}}+2\left(1+\sqrt{5}\right)\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\,\text{ArcTanh}\left[\frac{\sqrt{2-2\sqrt{1+x}}}{\sqrt{3+\sqrt{5}}}\right]+$$
$$\left(-1+\sqrt{5}\right)\sqrt{\frac{2}{5}\left(3+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{2}\sqrt{\frac{-1+\sqrt{1+x}}{-3+\sqrt{5}}}\right]+\frac{4\,\text{ArcTanh}\left[\frac{1+2\sqrt{1+x}}{\sqrt{5}}\right]}{\sqrt{5}}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}\,\left(1+x^2\right)}\,dx$$

Optimal (type 3, 365 leaves, 20 steps):

$$-\frac{i\,\text{ArcTan}\left[\frac{2+\sqrt{1-i}-\left(1-2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}}\right]}{2\sqrt{\frac{1-i}{i+\sqrt{1-i}}}}+\frac{i\,\text{ArcTan}\left[\frac{2+\sqrt{1+i}-\left(1-2\sqrt{1+i}\right)\sqrt{1+x}}{2\sqrt{-i+\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}}\right]}{2\sqrt{-\frac{1+i}{i-\sqrt{1+i}}}}+\frac{i\,\text{ArcTanh}\left[\frac{2-\sqrt{1-i}-\left(1+2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{-i+\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}}\right]}{2\sqrt{-\frac{1-i}{i-\sqrt{1-i}}}}-\frac{i\,\text{ArcTanh}\left[\frac{2-\sqrt{1+i}-\left(1+2\sqrt{1+i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}}\right]}{2\sqrt{\frac{1+i}{i+\sqrt{1+i}}}}$$

Result (type 3, 2177 leaves):

$$\frac{1}{2\sqrt{1-i}\,\sqrt{i-\sqrt{1-i}}}$$
$$i\left(-i+\sqrt{1-i}\right)\text{ArcTan}\Big[\Big(\left(-1-2\,i\right)+\left(2-4\,i\right)\sqrt{1-i}-\left(6-6\,i\right)\sqrt{1+x}-\left(1-2\,i\right)\sqrt{1-i}\,\sqrt{1+x}+4\,i\,\left(1+x\right)+\left(1+3\,i\right)\sqrt{1-i}\,\left(1+x\right)+$$
$$\left(4-4\,i\right)\sqrt{i-\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}-2\sqrt{1-i}\,\sqrt{i-\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}-\left(2-2\,i\right)\sqrt{i-\sqrt{1-i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}-$$

$$4\sqrt{1-i}\,\sqrt{i-\sqrt{1-i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}\Big)\Big/\left(1-(4-2i)\sqrt{1-i}-(2-2i)\sqrt{1+x}+\frac{4\sqrt{1+x}}{\sqrt{1-i}}+(6-4i)(1+x)+8\sqrt{1-i}\,(1+x)\right)\Big]+$$

$$\frac{1}{2\sqrt{1-i}}\,i\sqrt{i+\sqrt{1-i}}\,\mathrm{ArcTan}\Big[\Big((1+2i)+(2-4i)\sqrt{1-i}+(6-6i)\sqrt{1+x}-(1-2i)\sqrt{1-i}\,\sqrt{1+x}-4i(1+x)+(1+3i)\sqrt{1-i}\,(1+x)-$$

$$(4-4i)\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}-2\sqrt{1-i}\,\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}+(2-2i)\sqrt{i+\sqrt{1-i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}-$$

$$4\sqrt{1-i}\,\sqrt{i+\sqrt{1-i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}\Big)\Big/\left(-1-(4-2i)\sqrt{1-i}+(2-2i)\sqrt{1+x}+\frac{4\sqrt{1+x}}{\sqrt{1-i}}-(6-4i)(1+x)+8\sqrt{1-i}\,(1+x)\right)\Big]-$$

$$\frac{1}{2\sqrt{1+i}\,\sqrt{i-\sqrt{1+i}}}\left(-i+\sqrt{1+i}\right)\mathrm{ArcTan}\Big[\Big((-2-i)+(4-2i)\sqrt{1+i}+(6-6i)\sqrt{1+x}-(2-i)\sqrt{1+i}\,\sqrt{1+x}+4(1+x)-$$

$$(3+i)\sqrt{1+i}\,(1+x)+2i\sqrt{1+i}\,\sqrt{i-\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}+4i\sqrt{1+i}\,\sqrt{i-\sqrt{1+i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}\Big)\Big/$$

$$\left((-4+5i)+2\sqrt{1+i}+(2+6i)\sqrt{1+x}+(2+8i)\sqrt{1+i}\,\sqrt{1+x}+(3+3i)(1+x)+4i\sqrt{1+i}\,(1+x)\right)\Big]-$$

$$\frac{1}{2\sqrt{1+i}}\sqrt{i+\sqrt{1+i}}\,\mathrm{ArcTan}\Big[\Big((2+i)+(4-2i)\sqrt{1+i}-(6-6i)\sqrt{1+x}-(2-i)\sqrt{1+i}\,\sqrt{1+x}-4(1+x)-$$

$$(3+i)\sqrt{1+i}\,(1+x)+2i\sqrt{1+i}\,\sqrt{i+\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}+4i\sqrt{1+i}\,\sqrt{i+\sqrt{1+i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}\Big)\Big/$$

$$\left((4-5i)+2\sqrt{1+i}-(2+6i)\sqrt{1+x}+(2+8i)\sqrt{1+i}\,\sqrt{1+x}-(3+3i)(1+x)+4i\sqrt{1+i}\,(1+x)\right)\Big]-$$

$$\frac{\left(-i+\sqrt{1-i}\right)\mathrm{Log}\Big[\left(\sqrt{1-i}-\sqrt{1+x}\right)^2\Big]}{4\sqrt{1-i}\,\sqrt{i-\sqrt{1-i}}}-\frac{i\sqrt{i+\sqrt{1+i}}\,\mathrm{Log}\Big[\left(\sqrt{1+i}-\sqrt{1+x}\right)^2\Big]}{4\sqrt{1+i}}-$$

$$\frac{\sqrt{i+\sqrt{1-i}}\,\mathrm{Log}\Big[\left(\sqrt{1-i}+\sqrt{1+x}\right)^2\Big]}{4\sqrt{1-i}}-$$

$$\frac{i\left(-i+\sqrt{1+i}\right)\mathrm{Log}\Big[\left(\sqrt{1+i}+\sqrt{1+x}\right)^2\Big]}{4\sqrt{1+i}\,\sqrt{i-\sqrt{1+i}}}+$$

$$\frac{1}{4\sqrt{1-i}\,\sqrt{i-\sqrt{1-i}}}$$

$$\left(-i+\sqrt{1-i}\right)\mathrm{Log}\Big[(3+5i)+\frac{4}{\sqrt{1-i}}-8\sqrt{1+x}+(3-7i)\sqrt{1-i}\,\sqrt{1+x}-(8-5i)(1+x)-$$

$$\frac{4(1+x)}{\sqrt{1-i}}-2(1-i)^{3/2}\sqrt{i-\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}-4(1-i)^{3/2}\sqrt{i-\sqrt{1-i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}\Big]+$$

$$\frac{1}{4\sqrt{1-i}}\sqrt{i+\sqrt{1-i}}\;\text{Log}\Big[(-3-5i)+\frac{4}{\sqrt{1-i}}+8\sqrt{1+x}+(3-7i)\sqrt{1-i}\sqrt{1+x}+(8-5i)(1+x)-\frac{4(1+x)}{\sqrt{1-i}}-2(1-i)^{3/2}\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{1+x}}-4(1-i)^{3/2}\sqrt{i+\sqrt{1-i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}\Big]+\frac{1}{4\sqrt{1+i}\sqrt{i-\sqrt{1+i}}}$$

$$i\left(-i+\sqrt{1+i}\right)\text{Log}\Big[(-5+5i)-(6-2i)\sqrt{1+i}+(1+3i)\sqrt{1+i}\sqrt{1+x}-5(1+x)+(6-2i)\sqrt{1+i}(1+x)+8\sqrt{i-\sqrt{1+i}}\sqrt{x+\sqrt{1+x}}+\frac{4\sqrt{i-\sqrt{1+i}}\sqrt{x+\sqrt{1+x}}}{\sqrt{1+i}}-4\sqrt{i-\sqrt{1+i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}+\frac{8\sqrt{i-\sqrt{1+i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}{\sqrt{1+i}}\Big]+$$

$$\frac{1}{4\sqrt{1+i}}\,i\sqrt{i+\sqrt{1+i}}\;\text{Log}\Big[(5-5i)-(6-2i)\sqrt{1+i}+(1+3i)\sqrt{1+i}\sqrt{1+x}+5(1+x)+(6-2i)\sqrt{1+i}(1+x)-8\sqrt{i+\sqrt{1+i}}\sqrt{x+\sqrt{1+x}}+\frac{4\sqrt{i+\sqrt{1+i}}\sqrt{x+\sqrt{1+x}}}{\sqrt{1+i}}+4\sqrt{i+\sqrt{1+i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}+\frac{8\sqrt{i+\sqrt{1+i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}{\sqrt{1+i}}\Big]$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{x+\sqrt{1+x}}}{1+x^2}\,dx$$

Optimal (type 3, 337 leaves, 22 steps):

$$\frac{1}{2}i\sqrt{i+\sqrt{1-i}}\;\text{ArcTan}\Big[\frac{2+\sqrt{1-i}-\left(1-2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{1+x}}}\Big]-\frac{1}{2}i\sqrt{-i+\sqrt{1+i}}\;\text{ArcTan}\Big[\frac{2+\sqrt{1+i}-\left(1-2\sqrt{1+i}\right)\sqrt{1+x}}{2\sqrt{-i+\sqrt{1+i}}\sqrt{x+\sqrt{1+x}}}\Big]+$$

$$\frac{1}{2}i\sqrt{-i+\sqrt{1-i}}\;\text{ArcTanh}\Big[\frac{2-\sqrt{1-i}-\left(1+2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{-i+\sqrt{1-i}}\sqrt{x+\sqrt{1+x}}}\Big]-\frac{1}{2}i\sqrt{i+\sqrt{1+i}}\;\text{ArcTanh}\Big[\frac{2-\sqrt{1+i}-\left(1+2\sqrt{1+i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1+i}}\sqrt{x+\sqrt{1+x}}}\Big]$$

Result (type 3, 2581 leaves):

$$\frac{1}{2\sqrt{1-i}\sqrt{i-\sqrt{1-i}}}\left((1+i)+\sqrt{1-i}\right)$$

$$\text{ArcTan}\Big[\Big((2-3i)+(3-i)\sqrt{1-i}-8\sqrt{1+x}-5\sqrt{1-i}\sqrt{1+x}+(2+5i)(1+x)+5i\sqrt{1-i}(1+x)+4\sqrt{i-\sqrt{1-i}}\sqrt{x+\sqrt{1+x}}+$$

$$2\sqrt{1-i}\sqrt{i-\sqrt{1-i}}\sqrt{x+\sqrt{1+x}}-(6+2i)\sqrt{i-\sqrt{1-i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}-\frac{8\sqrt{i-\sqrt{1-i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}{\sqrt{1-i}}\Big)\Big/$$

$$\left(\left(-4+7\,i\right)-\left(6-2\,i\right)\sqrt{1-i}+\left(4-2\,i\right)\sqrt{1+x}+\left(6-2\,i\right)\sqrt{1-i}\,\sqrt{1+x}+\left(10+i\right)\left(1+x\right)+\left(8+4\,i\right)\sqrt{1-i}\,\left(1+x\right)\right)\Big]+$$

$$\frac{1}{2\sqrt{1-i}\,\sqrt{i+\sqrt{1-i}}}\left(\left(-1-i\right)+\sqrt{1-i}\right)$$

$$\text{ArcTan}\Big[\Big(\left(-2+3\,i\right)+\left(3-i\right)\sqrt{1-i}+8\sqrt{1+x}-5\sqrt{1-i}\,\sqrt{1+x}-\left(2+5\,i\right)\left(1+x\right)+5\,i\sqrt{1-i}\,\left(1+x\right)-4\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}+$$

$$2\sqrt{1-i}\,\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}+\left(6+2\,i\right)\sqrt{i+\sqrt{1-i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}-\frac{8\sqrt{i+\sqrt{1-i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}}{\sqrt{1-i}}\Big)\Big/$$

$$\left(\left(4-7\,i\right)-\left(6-2\,i\right)\sqrt{1-i}-\left(4-2\,i\right)\sqrt{1+x}+\left(6-2\,i\right)\sqrt{1-i}\,\sqrt{1+x}-\left(10+i\right)\left(1+x\right)+\left(8+4\,i\right)\sqrt{1-i}\,\left(1+x\right)\right)\Big]-$$

$$\frac{1}{2\sqrt{1+i}\,\sqrt{i-\sqrt{1+i}}}\,i\left(\left(-1+i\right)+\sqrt{1+i}\right)\text{ArcTan}\Big[\Big(\left(1+8\,i\right)-5\left(1+i\right)^{3/2}-\left(16+8\,i\right)\sqrt{1+x}+\left(10+5\,i\right)\sqrt{1+i}\,\sqrt{1+x}+$$

$$\left(9-8\,i\right)\left(1+x\right)-\left(5-10\,i\right)\sqrt{1+i}\,\left(1+x\right)-4\sqrt{i-\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}+\left(4-2\,i\right)\sqrt{1+i}\,\sqrt{i-\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}-$$

$$8\sqrt{i-\sqrt{1+i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}+\left(8-4\,i\right)\sqrt{1+i}\,\sqrt{i-\sqrt{1+i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}\Big)\Big/$$

$$\left(\left(9+20\,i\right)-12\left(1+i\right)^{3/2}-\left(14+20\,i\right)\sqrt{1+x}+\left(22+12\,i\right)\sqrt{1+i}\,\sqrt{1+x}+\left(6-15\,i\right)\left(1+x\right)+\left(2+12\,i\right)\sqrt{1+i}\,\left(1+x\right)\right)\Big]-$$

$$\frac{1}{2\sqrt{1+i}\,\sqrt{i+\sqrt{1+i}}}\,i\left(\left(1-i\right)+\sqrt{1+i}\right)\text{ArcTan}\Big[\Big(\left(-1-8\,i\right)-5\left(1+i\right)^{3/2}+\left(16+8\,i\right)\sqrt{1+x}+\left(10+5\,i\right)\sqrt{1+i}\,\sqrt{1+x}-$$

$$\left(9-8\,i\right)\left(1+x\right)-\left(5-10\,i\right)\sqrt{1+i}\,\left(1+x\right)+4\sqrt{i+\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}+\left(4-2\,i\right)\sqrt{1+i}\,\sqrt{i+\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}+$$

$$8\sqrt{i+\sqrt{1+i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}+\left(8-4\,i\right)\sqrt{1+i}\,\sqrt{i+\sqrt{1+i}}\,\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}\Big)\Big/$$

$$\left(\left(-9-20\,i\right)-12\left(1+i\right)^{3/2}+\left(14+20\,i\right)\sqrt{1+x}+\left(22+12\,i\right)\sqrt{1+i}\,\sqrt{1+x}-\left(6-15\,i\right)\left(1+x\right)+\left(2+12\,i\right)\sqrt{1+i}\,\left(1+x\right)\right)\Big]+$$

$$\frac{i\left(\left(1+i\right)+\sqrt{1-i}\right)\text{Log}\Big[\left(\sqrt{1-i}-\sqrt{1+x}\right)^2\Big]}{4\sqrt{1-i}\,\sqrt{i-\sqrt{1-i}}}+\frac{\left(\left(1-i\right)+\sqrt{1+i}\right)\text{Log}\Big[\left(\sqrt{1+i}-\sqrt{1+x}\right)^2\Big]}{4\sqrt{1+i}\,\sqrt{i+\sqrt{1+i}}}+$$

$$\frac{i\left(\left(-1-i\right)+\sqrt{1-i}\right)\text{Log}\Big[\left(\sqrt{1-i}+\sqrt{1+x}\right)^2\Big]}{4\sqrt{1-i}\,\sqrt{i+\sqrt{1-i}}}+$$

$$\frac{\left(\left(-1+i\right)+\sqrt{1+i}\right)\text{Log}\Big[\left(\sqrt{1+i}+\sqrt{1+x}\right)^2\Big]}{4\sqrt{1+i}\,\sqrt{i-\sqrt{1+i}}}-$$

$$\frac{1}{4\sqrt{1-i}\,\sqrt{i-\sqrt{1-i}}}$$

$$\mathrm{i}\left((1+\mathrm{i})+\sqrt{1-\mathrm{i}}\right)\mathrm{Log}\Big[(5+17\,\mathrm{i})+14\,\mathrm{i}\sqrt{1-\mathrm{i}}-(10+22\,\mathrm{i})\sqrt{1+x}+(5-19\,\mathrm{i})\sqrt{1-\mathrm{i}}\sqrt{1+x}-(25+2\,\mathrm{i})(1+x)-$$
$$(15+9\,\mathrm{i})\sqrt{1-\mathrm{i}}\,(1+x)-(4-4\,\mathrm{i})\sqrt{\mathrm{i}-\sqrt{1-\mathrm{i}}}\sqrt{x+\sqrt{1+x}}-(6-2\,\mathrm{i})\sqrt{1-\mathrm{i}}\sqrt{\mathrm{i}-\sqrt{1-\mathrm{i}}}\sqrt{x+\sqrt{1+x}}-$$
$$(8-8\,\mathrm{i})\sqrt{\mathrm{i}-\sqrt{1-\mathrm{i}}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}-(12-4\,\mathrm{i})\sqrt{1-\mathrm{i}}\sqrt{\mathrm{i}-\sqrt{1-\mathrm{i}}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}\Big]-$$
$$\frac{1}{4\sqrt{1-\mathrm{i}}\sqrt{\mathrm{i}+\sqrt{1-\mathrm{i}}}}\,\mathrm{i}\left((-1-\mathrm{i})+\sqrt{1-\mathrm{i}}\right)\mathrm{Log}\Big[(-5-17\,\mathrm{i})+14\,\mathrm{i}\sqrt{1-\mathrm{i}}+(10+22\,\mathrm{i})\sqrt{1+x}+(5-19\,\mathrm{i})\sqrt{1-\mathrm{i}}\sqrt{1+x}+$$
$$(25+2\,\mathrm{i})(1+x)-(15+9\,\mathrm{i})\sqrt{1-\mathrm{i}}\,(1+x)+(4-4\,\mathrm{i})\sqrt{\mathrm{i}+\sqrt{1-\mathrm{i}}}\sqrt{x+\sqrt{1+x}}-(6-2\,\mathrm{i})\sqrt{1-\mathrm{i}}\sqrt{\mathrm{i}+\sqrt{1-\mathrm{i}}}\sqrt{x+\sqrt{1+x}}+$$
$$(8-8\,\mathrm{i})\sqrt{\mathrm{i}+\sqrt{1-\mathrm{i}}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}-(12-4\,\mathrm{i})\sqrt{1-\mathrm{i}}\sqrt{\mathrm{i}+\sqrt{1-\mathrm{i}}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}\Big]-$$
$$\frac{1}{4\sqrt{1+\mathrm{i}}\sqrt{\mathrm{i}-\sqrt{1+\mathrm{i}}}}\left((-1+\mathrm{i})+\sqrt{1+\mathrm{i}}\right)\mathrm{Log}\Big[(-3+5\,\mathrm{i})-(2+4\,\mathrm{i})\sqrt{1+\mathrm{i}}+(2-2\,\mathrm{i})\sqrt{1+x}-(1-3\,\mathrm{i})\sqrt{1+\mathrm{i}}\sqrt{1+x}-$$
$$(8+7\,\mathrm{i})(1+x)+(9+3\,\mathrm{i})\sqrt{1+\mathrm{i}}\,(1+x)+(4+4\,\mathrm{i})\sqrt{\mathrm{i}-\sqrt{1+\mathrm{i}}}\sqrt{x+\sqrt{1+x}}-2(1+\mathrm{i})^{3/2}\sqrt{\mathrm{i}-\sqrt{1+\mathrm{i}}}\sqrt{x+\sqrt{1+x}}-$$
$$(8+4\,\mathrm{i})\sqrt{\mathrm{i}-\sqrt{1+\mathrm{i}}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}+8\sqrt{1+\mathrm{i}}\sqrt{\mathrm{i}-\sqrt{1+\mathrm{i}}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}\Big]-$$
$$\frac{1}{4\sqrt{1+\mathrm{i}}\sqrt{\mathrm{i}+\sqrt{1+\mathrm{i}}}}\left((1-\mathrm{i})+\sqrt{1+\mathrm{i}}\right)\mathrm{Log}\Big[(3-5\,\mathrm{i})-(2+4\,\mathrm{i})\sqrt{1+\mathrm{i}}-(2-2\,\mathrm{i})\sqrt{1+x}-(1-3\,\mathrm{i})\sqrt{1+\mathrm{i}}\sqrt{1+x}+$$
$$(8+7\,\mathrm{i})(1+x)+(9+3\,\mathrm{i})\sqrt{1+\mathrm{i}}\,(1+x)-(4+4\,\mathrm{i})\sqrt{\mathrm{i}+\sqrt{1+\mathrm{i}}}\sqrt{x+\sqrt{1+x}}-2(1+\mathrm{i})^{3/2}\sqrt{\mathrm{i}+\sqrt{1+\mathrm{i}}}\sqrt{x+\sqrt{1+x}}+$$
$$(8+4\,\mathrm{i})\sqrt{\mathrm{i}+\sqrt{1+\mathrm{i}}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}+8\sqrt{1+\mathrm{i}}\sqrt{\mathrm{i}+\sqrt{1+\mathrm{i}}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}}\Big]$$

Problem 15: Unable to integrate problem.

$$\int\sqrt{1+\sqrt{x}+\sqrt{1+2\sqrt{x}+2x}}\,dx$$

Optimal (type 2, 77 leaves, 2 steps):

$$\frac{2\sqrt{1+\sqrt{x}+\sqrt{1+2\sqrt{x}+2x}}\left(2+\sqrt{x}+6x^{3/2}-\left(2-\sqrt{x}\right)\sqrt{1+2\sqrt{x}+2x}\right)}{15\sqrt{x}}$$

Result (type 8, 29 leaves):

$$\int\sqrt{1+\sqrt{x}+\sqrt{1+2\sqrt{x}+2x}}\,dx$$

Problem 16: Unable to integrate problem.

$$\int \sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2+2\sqrt{2}\sqrt{x}+2x}}\,dx$$

Optimal (type 2, 118 leaves, 3 steps):

$$\frac{1}{15\sqrt{x}}2\sqrt{2}\sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2}\sqrt{1+\sqrt{2}\sqrt{x}+x}}\left(4+\sqrt{2}\sqrt{x}+3\sqrt{2}x^{3/2}-\sqrt{2}\left(2\sqrt{2}-\sqrt{x}\right)\sqrt{1+\sqrt{2}\sqrt{x}+x}\right)$$

Result (type 8, 38 leaves):

$$\int \sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2+2\sqrt{2}\sqrt{x}+2x}}\,dx$$

Problem 18: Unable to integrate problem.

$$\int \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,dx$$

Optimal (type 3, 96 leaves, 7 steps):

$$\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,x+\frac{1}{4}\text{ArcTan}\left[\frac{3+\sqrt{1+\frac{1}{x}}}{2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right]-\frac{3}{4}\text{ArcTanh}\left[\frac{1-3\sqrt{1+\frac{1}{x}}}{2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right]$$

Result (type 8, 19 leaves):

$$\int \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,dx$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+e^{-x}}}{-e^{-x}+e^{x}}\,dx$$

Optimal (type 3, 25 leaves, 6 steps):

$$-\sqrt{2}\ \text{ArcTanh}\left[\frac{\sqrt{1+e^{-x}}}{\sqrt{2}}\right]$$

Result (type 3, 112 leaves):

$$\frac{e^{x/2}\sqrt{1+e^{-x}}\ \left(\text{Log}\left[1-e^{x/2}\right]-\text{Log}\left[1+e^{x/2}\right]+\text{Log}\left[1-e^{x/2}+\sqrt{2}\ \sqrt{1+e^{x}}\ \right]-\text{Log}\left[1+e^{x/2}+\sqrt{2}\ \sqrt{1+e^{x}}\ \right]\right)}{\sqrt{2}\ \sqrt{1+e^{x}}}$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int\sqrt{1+e^{-x}}\ \text{Csch}[x]\ dx$$

Optimal (type 3, 25 leaves, 7 steps):

$$-2\sqrt{2}\ \text{ArcTanh}\left[\frac{\sqrt{1+e^{-x}}}{\sqrt{2}}\right]$$

Result (type 3, 126 leaves):

$$\frac{1}{\sqrt{1+e^{x}}}\sqrt{2}\ e^{x/2}\sqrt{1+e^{-x}}\ \left(\text{Log}\left[1-e^{-x/2}\right]+\text{Log}\left[1+e^{-x/2}\right]-\text{Log}\left[e^{-x/2}\left(-1+e^{x/2}+\sqrt{2}\ \sqrt{1+e^{x}}\right)\right]-\text{Log}\left[e^{-x/2}\left(1+e^{x/2}+\sqrt{2}\ \sqrt{1+e^{x}}\right)\right]\right)$$

Problem 21: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(\text{Cos}[x]+\text{Cos}[3x]\right)^5}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\frac{523}{256}\text{ArcTanh}[\text{Sin}[x]]+\frac{1483\ \text{ArcTanh}\left[\sqrt{2}\ \text{Sin}[x]\right]}{512\sqrt{2}}+\frac{\text{Sin}[x]}{32\left(1-2\,\text{Sin}[x]^2\right)^4}-\frac{17\,\text{Sin}[x]}{192\left(1-2\,\text{Sin}[x]^2\right)^3}+\frac{203\,\text{Sin}[x]}{768\left(1-2\,\text{Sin}[x]^2\right)^2}-\frac{437\,\text{Sin}[x]}{512\left(1-2\,\text{Sin}[x]^2\right)}-\frac{43}{256}\text{Sec}[x]\ \text{Tan}[x]-\frac{1}{128}\text{Sec}[x]^3\ \text{Tan}[x]$$

Result (type 3, 478 leaves):

$$-\frac{1483\, i\, \text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]-\sqrt{2}\,\text{Sin}\left[\frac{x}{2}\right]}{-\text{Cos}\left[\frac{x}{2}\right]+\sqrt{2}\,\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}\right]}{1024\sqrt{2}}+\frac{\left(\frac{1483}{2048}+\frac{1483\, i}{2048}\right)\left((-1-i)+\sqrt{2}\right)\text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]-\sqrt{2}\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\sqrt{2}\,\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}\right]}{(-1+i)+\sqrt{2}}+$$

$$\frac{523}{256}\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]-\frac{523}{256}\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1483\,\text{Log}\left[\sqrt{2}+2\,\text{Sin}[x]\right]}{1024\sqrt{2}}-\frac{1483\,\text{Log}\left[2-\sqrt{2}\,\text{Cos}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+$$

$$\frac{\left(\frac{1483}{4096}-\frac{1483\, i}{4096}\right)\left((-1-i)+\sqrt{2}\right)\text{Log}\left[2+\sqrt{2}\,\text{Cos}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{(-1+i)+\sqrt{2}}-\frac{1}{512\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^4}-\frac{43}{512\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^2}+$$

$$\frac{1}{512\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^4}+\frac{43}{512\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2}-\frac{17}{768\,(\text{Cos}[x]-\text{Sin}[x])^3}-\frac{437}{1024\,(\text{Cos}[x]-\text{Sin}[x])}+\frac{\text{Sin}[x]}{128\,(\text{Cos}[x]-\text{Sin}[x])^4}+$$

$$\frac{83\,\text{Sin}[x]}{512\,(\text{Cos}[x]-\text{Sin}[x])^2}+\frac{\text{Sin}[x]}{128\,(\text{Cos}[x]+\text{Sin}[x])^4}+\frac{17}{768\,(\text{Cos}[x]+\text{Sin}[x])^3}+\frac{83\,\text{Sin}[x]}{512\,(\text{Cos}[x]+\text{Sin}[x])^2}+\frac{437}{1024\,(\text{Cos}[x]+\text{Sin}[x])}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{\sqrt{e^x+e^{2x}}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$2\,e^{-x}\sqrt{e^x+e^{2x}}-\frac{\text{ArcTan}\left[\frac{i-(1-2\,i)\,e^x}{2\sqrt{1+i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1+i}}+\frac{\text{ArcTan}\left[\frac{i+(1+2\,i)\,e^x}{2\sqrt{1-i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1-i}}$$

Result (type 3, 444 leaves) :

$$\frac{1}{2\sqrt{e^x\,(1+e^x)}}$$

$$\Big(4+4\,e^x+(1+i)^{3/2}\,e^{x/2}\sqrt{1+e^x}\,\text{Log}\left[(-1)^{1/4}-e^{-x/2}\right]+(1-i)^{3/2}\,e^{x/2}\sqrt{1+e^x}\,\text{Log}\left[-(-1)^{3/4}-e^{-x/2}\right]+(1+i)^{3/2}\,e^{x/2}\sqrt{1+e^x}\,\text{Log}\left[(-1)^{1/4}+e^{-x/2}\right]+$$

$$(1-i)^{3/2}\,e^{x/2}\sqrt{1+e^x}\,\text{Log}\left[-(-1)^{3/4}+e^{-x/2}\right]-(1-i)^{3/2}\,e^{x/2}\sqrt{1+e^x}\,\text{Log}\left[e^{-x/2}\left(-(-1)^{3/4}+e^{x/2}+\sqrt{1-i}\,\sqrt{1+e^x}\right)\right]-$$

$$(1-i)^{3/2}\,e^{x/2}\sqrt{1+e^x}\,\text{Log}\left[e^{-x/2}\left((-1)^{3/4}+e^{x/2}+\sqrt{1-i}\,\sqrt{1+e^x}\right)\right]-(1+i)^{3/2}\,e^{x/2}\sqrt{1+e^x}\,\text{Log}\left[e^{-x/2}\left(-(-1)^{1/4}+e^{x/2}+\sqrt{1+i}\,\sqrt{1+e^x}\right)\right]-$$

$$(1+i)^{3/2}\,e^{x/2}\sqrt{1+e^x}\,\text{Log}\left[e^{-x/2}\left((-1)^{1/4}+e^{x/2}+\sqrt{1+i}\,\sqrt{1+e^x}\right)\right]\Big)$$

Problem 26: Unable to integrate problem.

$$\int \text{Log}\left[x^2+\sqrt{1-x^2}\right] dx$$

Optimal (type 3, 185 leaves, ? steps):

$$-2x-\text{ArcSin}[x]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]+$$
$$\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]+x\,\text{Log}\left[x^2+\sqrt{1-x^2}\right]$$

Result (type 8, 18 leaves) :

$$\int \text{Log}\left[x^2+\sqrt{1-x^2}\right] dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{\text{Log}\left[1+e^x\right]}{1+e^{2x}} dx$$

Optimal (type 4, 102 leaves, 12 steps) :

$$-\frac{1}{2}\text{Log}\left[\left(\frac{1}{2}-\frac{i}{2}\right)\left(i-e^x\right)\right]\text{Log}\left[1+e^x\right]-\frac{1}{2}\text{Log}\left[\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i+e^x\right)\right]\text{Log}\left[1+e^x\right]-$$
$$\text{PolyLog}\left[2,-e^x\right]-\frac{1}{2}\text{PolyLog}\left[2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+e^x\right)\right]-\frac{1}{2}\text{PolyLog}\left[2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+e^x\right)\right]$$

Result (type 8, 18 leaves) :

$$\int \frac{\text{Log}\left[1+e^x\right]}{1+e^{2x}} dx$$

Problem 28: Unable to integrate problem.

$$\int \text{Cosh}[x]\ \text{Log}\left[1+\text{Cosh}[x]^2\right]^2 dx$$

Optimal (type 4, 159 leaves, 13 steps) :

$$-8\sqrt{2}\,\text{ArcTan}\left[\frac{\text{Sinh}[x]}{\sqrt{2}}\right]+4\,i\,\sqrt{2}\,\text{ArcTan}\left[\frac{\text{Sinh}[x]}{\sqrt{2}}\right]^2+8\sqrt{2}\,\text{ArcTan}\left[\frac{\text{Sinh}[x]}{\sqrt{2}}\right]\text{Log}\left[\frac{2\sqrt{2}}{\sqrt{2}+i\,\text{Sinh}[x]}\right]+4\sqrt{2}\,\text{ArcTan}\left[\frac{\text{Sinh}[x]}{\sqrt{2}}\right]\text{Log}\left[2+\text{Sinh}[x]^2\right]+$$

$$4\,i\,\sqrt{2}\,\text{PolyLog}\left[2,1-\frac{2\sqrt{2}}{\sqrt{2}+i\,\text{Sinh}[x]}\right]+8\,\text{Sinh}[x]-4\,\text{Log}\left[2+\text{Sinh}[x]^2\right]\text{Sinh}[x]+\text{Log}\left[2+\text{Sinh}[x]^2\right]^2\text{Sinh}[x]$$

Result (type 8, 14 leaves):

$$\int \text{Cosh}[x]\,\text{Log}\left[1+\text{Cosh}[x]^2\right]^2\,dx$$

Problem 29: Unable to integrate problem.

$$\int \text{Cosh}[x]\,\text{Log}\left[\text{Cosh}[x]^2+\text{Sinh}[x]\right]^2\,dx$$

Optimal (type 4, 395 leaves, 28 steps):

$$-4\sqrt{3}\,\text{ArcTan}\left[\frac{1+2\,\text{Sinh}[x]}{\sqrt{3}}\right]-\frac{1}{2}\left(1-i\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2\,\text{Sinh}[x]\right]^2-\left(1+i\sqrt{3}\right)\text{Log}\left[\frac{i\left(1-i\sqrt{3}+2\,\text{Sinh}[x]\right)}{2\sqrt{3}}\right]\text{Log}\left[1+i\sqrt{3}+2\,\text{Sinh}[x]\right]-$$

$$\frac{1}{2}\left(1+i\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}+2\,\text{Sinh}[x]\right]^2-\left(1-i\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2\,\text{Sinh}[x]\right]\text{Log}\left[-\frac{i\left(1+i\sqrt{3}+2\,\text{Sinh}[x]\right)}{2\sqrt{3}}\right]-$$

$$2\,\text{Log}\left[1+\text{Sinh}[x]+\text{Sinh}[x]^2\right]+\left(1-i\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2\,\text{Sinh}[x]\right]\text{Log}\left[1+\text{Sinh}[x]+\text{Sinh}[x]^2\right]+$$

$$\left(1+i\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}+2\,\text{Sinh}[x]\right]\text{Log}\left[1+\text{Sinh}[x]+\text{Sinh}[x]^2\right]-\left(1+i\sqrt{3}\right)\text{PolyLog}\left[2,-\frac{i-\sqrt{3}+2\,i\,\text{Sinh}[x]}{2\sqrt{3}}\right]-$$

$$\left(1-i\sqrt{3}\right)\text{PolyLog}\left[2,\frac{i+\sqrt{3}+2\,i\,\text{Sinh}[x]}{2\sqrt{3}}\right]+8\,\text{Sinh}[x]-4\,\text{Log}\left[1+\text{Sinh}[x]+\text{Sinh}[x]^2\right]\text{Sinh}[x]+\text{Log}\left[1+\text{Sinh}[x]+\text{Sinh}[x]^2\right]^2\text{Sinh}[x]$$

Result (type 8, 15 leaves):

$$\int \text{Cosh}[x]\,\text{Log}\left[\text{Cosh}[x]^2+\text{Sinh}[x]\right]^2\,dx$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Log}\left[x+\sqrt{1+x}\right]^2}{(1+x)^2}\,dx$$

Optimal (type 4, 555 leaves, 35 steps):

$$\text{Log}[1+x]+\frac{2\,\text{Log}\left[x+\sqrt{1+x}\right]}{\sqrt{1+x}}-6\,\text{Log}\left[\sqrt{1+x}\right]\,\text{Log}\left[x+\sqrt{1+x}\right]-\frac{\text{Log}\left[x+\sqrt{1+x}\right]^2}{1+x}-\left(1+\sqrt{5}\right)\,\text{Log}\left[1-\sqrt{5}+2\,\sqrt{1+x}\right]+$$
$$6\,\text{Log}\left[\frac{1}{2}\left(-1+\sqrt{5}\right)\right]\,\text{Log}\left[1-\sqrt{5}+2\,\sqrt{1+x}\right]+\left(3+\sqrt{5}\right)\,\text{Log}\left[x+\sqrt{1+x}\right]\,\text{Log}\left[1-\sqrt{5}+2\,\sqrt{1+x}\right]-$$
$$\frac{1}{2}\left(3+\sqrt{5}\right)\,\text{Log}\left[1-\sqrt{5}+2\,\sqrt{1+x}\right]^2-\left(1-\sqrt{5}\right)\,\text{Log}\left[1+\sqrt{5}+2\,\sqrt{1+x}\right]+\left(3-\sqrt{5}\right)\,\text{Log}\left[x+\sqrt{1+x}\right]\,\text{Log}\left[1+\sqrt{5}+2\,\sqrt{1+x}\right]-$$
$$\left(3-\sqrt{5}\right)\,\text{Log}\left[-\frac{1-\sqrt{5}+2\,\sqrt{1+x}}{2\,\sqrt{5}}\right]\,\text{Log}\left[1+\sqrt{5}+2\,\sqrt{1+x}\right]-\frac{1}{2}\left(3-\sqrt{5}\right)\,\text{Log}\left[1+\sqrt{5}+2\,\sqrt{1+x}\right]^2-$$
$$\left(3+\sqrt{5}\right)\,\text{Log}\left[1-\sqrt{5}+2\,\sqrt{1+x}\right]\,\text{Log}\left[\frac{1+\sqrt{5}+2\,\sqrt{1+x}}{2\,\sqrt{5}}\right]+6\,\text{Log}\left[\sqrt{1+x}\right]\,\text{Log}\left[1+\frac{2\,\sqrt{1+x}}{1+\sqrt{5}}\right]+6\,\text{PolyLog}\left[2,-\frac{2\,\sqrt{1+x}}{1+\sqrt{5}}\right]-$$
$$\left(3+\sqrt{5}\right)\,\text{PolyLog}\left[2,-\frac{1-\sqrt{5}+2\,\sqrt{1+x}}{2\,\sqrt{5}}\right]-\left(3-\sqrt{5}\right)\,\text{PolyLog}\left[2,\frac{1+\sqrt{5}+2\,\sqrt{1+x}}{2\,\sqrt{5}}\right]-6\,\text{PolyLog}\left[2,1+\frac{2\,\sqrt{1+x}}{1-\sqrt{5}}\right]$$

Result (type 4, 1283 leaves) :

$$\text{Log}[1+x]-\text{Log}\left[-1+\sqrt{5}-2\sqrt{1+x}\right]-\sqrt{5}\,\text{Log}\left[-1+\sqrt{5}-2\sqrt{1+x}\right]+\frac{\text{Log}[100]\,\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]}{\sqrt{5}}-6\,\text{Log}\left[\frac{2\sqrt{1+x}}{-1+\sqrt{5}}\right]\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]+$$

$$3\,\text{Log}[1+x]\,\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]-3\,\text{Log}\left[-1+\sqrt{5}-2\sqrt{1+x}\right]\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]-\sqrt{5}\,\text{Log}\left[-1+\sqrt{5}-2\sqrt{1+x}\right]\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]+$$

$$\frac{3}{2}\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]^2+\frac{1}{2}\sqrt{5}\,\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]^2+\frac{\text{Log}[8]\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]}{2\sqrt{5}}-$$

$$3\,\text{Log}\left[-1+\sqrt{5}-2\sqrt{1+x}\right]\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]-\sqrt{5}\,\text{Log}\left[-1+\sqrt{5}-2\sqrt{1+x}\right]\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]+$$

$$\frac{3}{2}\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]^2-\frac{\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]^2}{\sqrt{5}}+\frac{2\,\text{Log}\left[x+\sqrt{1+x}\right]}{\sqrt{1+x}}-3\,\text{Log}[1+x]\,\text{Log}\left[x+\sqrt{1+x}\right]+$$

$$3\,\text{Log}\left[-1+\sqrt{5}-2\sqrt{1+x}\right]\text{Log}\left[x+\sqrt{1+x}\right]+\sqrt{5}\,\text{Log}\left[-1+\sqrt{5}-2\sqrt{1+x}\right]\text{Log}\left[x+\sqrt{1+x}\right]-\frac{\text{Log}\left[x+\sqrt{1+x}\right]^2}{1+x}-\text{Log}\left[1+\sqrt{5}+2\sqrt{1+x}\right]+$$

$$\sqrt{5}\,\text{Log}\left[1+\sqrt{5}+2\sqrt{1+x}\right]-3\,\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]\text{Log}\left[1+\sqrt{5}+2\sqrt{1+x}\right]+\sqrt{5}\,\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]\text{Log}\left[1+\sqrt{5}+2\sqrt{1+x}\right]-$$

$$3\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]\text{Log}\left[1+\sqrt{5}+2\sqrt{1+x}\right]+\frac{7\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]\text{Log}\left[1+\sqrt{5}+2\sqrt{1+x}\right]}{2\sqrt{5}}+$$

$$3\,\text{Log}\left[x+\sqrt{1+x}\right]\text{Log}\left[1+\sqrt{5}+2\sqrt{1+x}\right]-\sqrt{5}\,\text{Log}\left[x+\sqrt{1+x}\right]\text{Log}\left[1+\sqrt{5}+2\sqrt{1+x}\right]+3\,\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]\text{Log}\left[\frac{1+\sqrt{5}+2\sqrt{1+x}}{2\sqrt{5}}\right]-$$

$$\frac{3\,\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]\text{Log}\left[\frac{1+\sqrt{5}+2\sqrt{1+x}}{2\sqrt{5}}\right]}{\sqrt{5}}+3\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]\text{Log}\left[\frac{1}{10}\left(5-\sqrt{5}-2\sqrt{5}\sqrt{1+x}\right)\right]+$$

$$\sqrt{5}\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{5}\right)+\sqrt{1+x}\right]\text{Log}\left[\frac{1}{10}\left(5-\sqrt{5}-2\sqrt{5}\sqrt{1+x}\right)\right]-\frac{2\,\text{Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]\text{Log}\left[5+\sqrt{5}+2\sqrt{5}\sqrt{1+x}\right]}{\sqrt{5}}+$$

$$3\,\text{Log}[1+x]\,\text{Log}\left[1+\frac{2\sqrt{1+x}}{1+\sqrt{5}}\right]+6\,\text{PolyLog}\left[2,-\frac{2\sqrt{1+x}}{1+\sqrt{5}}\right]-\left(-3+\sqrt{5}\right)\text{PolyLog}\left[2,\frac{-1+\sqrt{5}-2\sqrt{1+x}}{2\sqrt{5}}\right]-$$

$$6\,\text{PolyLog}\left[2,\frac{-1+\sqrt{5}-2\sqrt{1+x}}{-1+\sqrt{5}}\right]+3\,\text{PolyLog}\left[2,\frac{1+\sqrt{5}+2\sqrt{1+x}}{2\sqrt{5}}\right]+\sqrt{5}\,\text{PolyLog}\left[2,\frac{1+\sqrt{5}+2\sqrt{1+x}}{2\sqrt{5}}\right]$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \text{ArcTan}[2\,\text{Tan}[x]]\,dx$$

Optimal (type 4, 80 leaves, 7 steps):

$$x\,\text{ArcTan}[2\,\text{Tan}[x]]+\frac{1}{2}\,i\,x\,\text{Log}\left[1-3\,e^{2\,i\,x}\right]-\frac{1}{2}\,i\,x\,\text{Log}\left[1-\frac{1}{3}\,e^{2\,i\,x}\right]-\frac{1}{4}\,\text{PolyLog}\left[2,\,\frac{1}{3}\,e^{2\,i\,x}\right]+\frac{1}{4}\,\text{PolyLog}\left[2,\,3\,e^{2\,i\,x}\right]$$

Result (type 4, 262 leaves):

$$x\,\text{ArcTan}[2\,\text{Tan}[x]]-$$
$$\frac{1}{4}\,i\left(4\,i\,x\,\text{ArcTan}\left[\frac{\text{Cot}[x]}{2}\right]+2\,i\,\text{ArcCos}\left[\frac{5}{3}\right]\,\text{ArcTan}[2\,\text{Tan}[x]]+\left(\text{ArcCos}\left[\frac{5}{3}\right]+2\,\text{ArcTan}\left[\frac{\text{Cot}[x]}{2}\right]+2\,\text{ArcTan}[2\,\text{Tan}[x]]\right)\,\text{Log}\left[\frac{2\,i\,\sqrt{\frac{2}{3}}\,e^{-i\,x}}{\sqrt{-5+3\,\text{Cos}[2\,x]}}\right]+\right.$$
$$\left(\text{ArcCos}\left[\frac{5}{3}\right]-2\,\text{ArcTan}\left[\frac{\text{Cot}[x]}{2}\right]-2\,\text{ArcTan}[2\,\text{Tan}[x]]\right)\,\text{Log}\left[\frac{2\,i\,\sqrt{\frac{2}{3}}\,e^{i\,x}}{\sqrt{-5+3\,\text{Cos}[2\,x]}}\right]-\left(\text{ArcCos}\left[\frac{5}{3}\right]-2\,\text{ArcTan}[2\,\text{Tan}[x]]\right)\,\text{Log}\left[\frac{4\,i-4\,\text{Tan}[x]}{i+2\,\text{Tan}[x]}\right]-$$
$$\left.\left(\text{ArcCos}\left[\frac{5}{3}\right]+2\,\text{ArcTan}[2\,\text{Tan}[x]]\right)\,\text{Log}\left[\frac{4\,(i+\text{Tan}[x])}{3\,i+6\,\text{Tan}[x]}\right]+i\left(-\text{PolyLog}\left[2,\,\frac{-3\,i+6\,\text{Tan}[x]}{i+2\,\text{Tan}[x]}\right]+\text{PolyLog}\left[2,\,\frac{-i+2\,\text{Tan}[x]}{3\,i+6\,\text{Tan}[x]}\right]\right)\right)$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int\sqrt{1+x^2}\,\text{ArcTan}[x]^2\,dx$$

Optimal (type 4, 121 leaves, 10 steps):

$$\text{ArcSinh}[x]-\sqrt{1+x^2}\,\text{ArcTan}[x]+\frac{1}{2}\,x\,\sqrt{1+x^2}\,\text{ArcTan}[x]^2-i\,\text{ArcTan}\left[e^{i\,\text{ArcTan}[x]}\right]\,\text{ArcTan}[x]^2+$$
$$i\,\text{ArcTan}[x]\,\text{PolyLog}\left[2,\,-i\,e^{i\,\text{ArcTan}[x]}\right]-i\,\text{ArcTan}[x]\,\text{PolyLog}\left[2,\,i\,e^{i\,\text{ArcTan}[x]}\right]-\text{PolyLog}\left[3,\,-i\,e^{i\,\text{ArcTan}[x]}\right]+\text{PolyLog}\left[3,\,i\,e^{i\,\text{ArcTan}[x]}\right]$$

Result (type 4, 405 leaves):

$$\frac{1}{2}\left(\sqrt{1+x^2}\ \text{ArcTan}[x]\ (-2+x\,\text{ArcTan}[x]) - \pi\,\text{ArcTan}[x]\ \text{Log}[2] + \text{ArcTan}[x]^2\ \text{Log}\left[1-i\,e^{i\,\text{ArcTan}[x]}\right] - \text{ArcTan}[x]^2\ \text{Log}\left[1+i\,e^{i\,\text{ArcTan}[x]}\right] + \right.$$
$$\pi\,\text{ArcTan}[x]\ \text{Log}\left[\left(-\frac{1}{2}-\frac{i}{2}\right) e^{-\frac{1}{2} i\,\text{ArcTan}[x]} \left(-i+e^{i\,\text{ArcTan}[x]}\right)\right] - \text{ArcTan}[x]^2\ \text{Log}\left[\left(\frac{1}{2}+\frac{i}{2}\right) e^{-\frac{1}{2} i\,\text{ArcTan}[x]} \left(-i+e^{i\,\text{ArcTan}[x]}\right)\right] +$$
$$\pi\,\text{ArcTan}[x]\ \text{Log}\left[\frac{1}{2} e^{-\frac{1}{2} i\,\text{ArcTan}[x]} \left((1+i)+(1-i)\,e^{i\,\text{ArcTan}[x]}\right)\right] + \text{ArcTan}[x]^2\ \text{Log}\left[\frac{1}{2} e^{-\frac{1}{2} i\,\text{ArcTan}[x]} \left((1+i)+(1-i)\,e^{i\,\text{ArcTan}[x]}\right)\right] -$$
$$\pi\,\text{ArcTan}[x]\ \text{Log}\left[-\text{Cos}\left[\frac{1}{4}(\pi+2\,\text{ArcTan}[x])\right]\right] - 2\,\text{Log}\left[\text{Cos}\left[\frac{\text{ArcTan}[x]}{2}\right]-\text{Sin}\left[\frac{\text{ArcTan}[x]}{2}\right]\right] +$$
$$\text{ArcTan}[x]^2\ \text{Log}\left[\text{Cos}\left[\frac{\text{ArcTan}[x]}{2}\right]-\text{Sin}\left[\frac{\text{ArcTan}[x]}{2}\right]\right] + 2\,\text{Log}\left[\text{Cos}\left[\frac{\text{ArcTan}[x]}{2}\right]+\text{Sin}\left[\frac{\text{ArcTan}[x]}{2}\right]\right] -$$
$$\text{ArcTan}[x]^2\ \text{Log}\left[\text{Cos}\left[\frac{\text{ArcTan}[x]}{2}\right]+\text{Sin}\left[\frac{\text{ArcTan}[x]}{2}\right]\right] - \pi\,\text{ArcTan}[x]\ \text{Log}\left[\text{Sin}\left[\frac{1}{4}(\pi+2\,\text{ArcTan}[x])\right]\right] +$$
$$\left. 2\,i\,\text{ArcTan}[x]\ \text{PolyLog}\left[2,-i\,e^{i\,\text{ArcTan}[x]}\right] - 2\,i\,\text{ArcTan}[x]\ \text{PolyLog}\left[2,i\,e^{i\,\text{ArcTan}[x]}\right] - 2\,\text{PolyLog}\left[3,-i\,e^{i\,\text{ArcTan}[x]}\right] + 2\,\text{PolyLog}\left[3,i\,e^{i\,\text{ArcTan}[x]}\right]\right)$$

Test results for the 14 problems in "Bronstein Problems.m"

Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 252 leaves, 3 steps):

$$\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{3^{1/4}\sqrt{2-\sqrt{3}}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\ \sqrt{1-x^3}} +$$
$$\frac{2\sqrt{2}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\ \sqrt{1-x^3}}$$

Result (type 4, 122 leaves):

$$\frac{1}{3^{1/4}\sqrt{1-x^3}} 2\,(-1)^{1/6}\sqrt{(-1)^{5/6}\,(-1+x)}\,\sqrt{1+x+x^2}$$
$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 6: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{-71-96\,x+10\,x^2+x^4}}\,dx$$

Optimal (type 3, 76 leaves, 1 step):

$$\frac{1}{8}\,\text{Log}\left[10\,001+3124\,x^2-1408\,x^3+54\,x^4-128\,x^5+20\,x^6+x^8+\sqrt{-71-96\,x+10\,x^2+x^4}\,\left(781-528\,x+27\,x^2-80\,x^3+15\,x^4+x^6\right)\right]$$

Result (type 4, 1226 leaves):

$$-\left(\left(2\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\right.\right.$$

$$\left(\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\right.\right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.4\right]\right)\right)\Big/\left(\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right)\right)\right],$$

$$\left(\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,3\right]\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right)\Big/$$

$$\left(\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,3\right]\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right)\right]-$$

$$4\sqrt{2\left(-1+\sqrt{3}\right)}\ \text{EllipticPi}\left[\frac{\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]}{\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]},\right.$$

$$\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right)\Big/\right.}\right.$$

$$\left.\left.\left(\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right)\right)}\right],$$

$$\left(\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,3\right]\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right)\Big/$$

$$\left.\left(\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,3\right]\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right)\right]\Bigg)$$

$$\sqrt{\frac{x-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,3\right]}{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,3\right]\right)}}$$

$$\sqrt{\frac{\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)}{\left(\sqrt{3}+2\sqrt{2\left(-1+\sqrt{3}\right)}-x\right)\left(\sqrt{3}-2\sqrt{2\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)}}$$

$$\left.\left(x-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right)\Big/$$

$$\left(\sqrt{-71-96\,x+10\,x^2+x^4}\,\left(\sqrt{3}+2\,\sqrt{2\,\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)\right.$$

$$\left.\left.\sqrt{\frac{x-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]}{\left(\sqrt{3}+2\,\sqrt{2\,\left(-1+\sqrt{3}\right)}-x\right)\,\left(\sqrt{3}-2\,\sqrt{2\,\left(-1+\sqrt{3}\right)}-\text{Root}\left[-71-96\,\#1+10\,\#1^2+\#1^4\,\&,\,4\right]\right)}}\right)\right)$$

Test results for the 50 problems in "Charlwood Problems.m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int -\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]\,dx$$

Optimal (type 3, 69 leaves, ? steps):

$$\frac{\left(\sqrt{x}+3\,\sqrt{1+x}\right)\,\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{4\,\sqrt{2}}-\left(\frac{3}{8}+x\right)\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]$$

Result (type 3, 205 leaves):

$$-x\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]-\left(\left(1+x\right)\,\left(1+2\,x-2\,\sqrt{x}\,\sqrt{1+x}\right)^2\right.$$

$$\left.\left(2\,\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}\,\left(-3-2\,x+2\,\sqrt{x}\,\sqrt{1+x}\right)+3\,\sqrt{-2-4\,x+4\,\sqrt{x}\,\sqrt{1+x}}\,\text{Log}\left[2\,\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}+\sqrt{-2-4\,x+4\,\sqrt{x}\,\sqrt{1+x}}\right]\right)\right)\Big/$$

$$\left(8\,\sqrt{2}\,\left(-\sqrt{x}+\sqrt{1+x}\right)^3\,\left(1+x-\sqrt{x}\,\sqrt{1+x}\right)^2\right)$$

Problem 5: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x]^2}{\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\,dx$$

Optimal (type 3, 45 leaves, ? steps):

$$\frac{x}{3}+\frac{1}{3}\,\text{ArcTan}\left[\frac{\text{Cos}[x]\,\left(1+\text{Cos}[x]^2\right)\,\text{Sin}[x]}{1+\text{Cos}[x]^2\,\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\right]$$

Result (type 4, 159 leaves) :

$$-\left(\left(2\,\mathrm{i}\,\text{Cos}[x]^2\,\text{EllipticPi}\left[\frac{3}{2}+\frac{\mathrm{i}\,\sqrt{3}}{2},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{2\,\mathrm{i}}{-3\,\mathrm{i}+\sqrt{3}}}\,\text{Tan}[x]\right],\,\frac{3\,\mathrm{i}-\sqrt{3}}{3\,\mathrm{i}+\sqrt{3}}\right]\sqrt{1-\frac{2\,\mathrm{i}\,\text{Tan}[x]^2}{-3\,\mathrm{i}+\sqrt{3}}}\,\sqrt{1+\frac{2\,\mathrm{i}\,\text{Tan}[x]^2}{3\,\mathrm{i}+\sqrt{3}}}\right)\Big/\left(\sqrt{-\frac{\mathrm{i}}{-3\,\mathrm{i}+\sqrt{3}}}\,\sqrt{15+8\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}\right)\right)$$

Problem 6: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Tan}[x]\,\sqrt{1+\text{Tan}[x]^4}\,\mathrm{d}x$$

Optimal (type 3, 56 leaves, 7 steps) :

$$-\frac{1}{2}\,\text{ArcSinh}\left[\text{Tan}[x]^2\right]-\frac{\text{ArcTanh}\left[\frac{1-\text{Tan}[x]^2}{\sqrt{2}\,\sqrt{1+\text{Tan}[x]^4}}\right]}{\sqrt{2}}+\frac{1}{2}\,\sqrt{1+\text{Tan}[x]^4}$$

Result (type 4, 52 283 leaves) : Display of huge result suppressed!

Problem 7: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[x]}{\sqrt{1+\text{Sec}[x]^3}}\,\mathrm{d}x$$

Optimal (type 3, 15 leaves, 4 steps) :

$$-\frac{2}{3}\,\text{ArcTanh}\left[\sqrt{1+\text{Sec}[x]^3}\right]$$

Result (type 4, 3292 leaves) :

$$-\left(\left(\mathrm{i}\,\mathrm{Cos}[x]^2\left(\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{3}\sqrt{\frac{\mathrm{i}\,\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\right],\frac{3\,\mathrm{i}-\sqrt{3}}{3\,\mathrm{i}+\sqrt{3}}\right]-\right.\right.\right.$$

$$\left.\mathrm{EllipticPi}\left[\frac{1}{6}\left(3+\mathrm{i}\sqrt{3}\right),\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{3}\sqrt{\frac{\mathrm{i}\,\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\right],\frac{3\,\mathrm{i}-\sqrt{3}}{3\,\mathrm{i}+\sqrt{3}}\right]\right)\mathrm{Sec}\left[\frac{x}{2}\right]^4\sqrt{\left(4+3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]\right)\mathrm{Sec}[x]^3}$$

$$\left(-\frac{\sqrt{\frac{4}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}+\frac{3\,\mathrm{Cos}[x]}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}+\frac{\mathrm{Cos}[3\,x]}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}}\,\mathrm{Sec}\left[\frac{x}{2}\right]\mathrm{Sin}\left[\frac{3\,x}{2}\right]}{2\left(3-2\,\mathrm{Cos}[x]+\mathrm{Cos}[2\,x]\right)}+\frac{\sqrt{\frac{4}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}+\frac{3\,\mathrm{Cos}[x]}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}+\frac{\mathrm{Cos}[3\,x]}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}}\,\mathrm{Sec}\left[\frac{x}{2}\right]\mathrm{Sin}\left[\frac{5\,x}{2}\right]}{2\left(3-2\,\mathrm{Cos}[x]+\mathrm{Cos}[2\,x]\right)}+\right.$$

$$\left.\left.\frac{\sqrt{\frac{4}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}+\frac{3\,\mathrm{Cos}[x]}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}+\frac{\mathrm{Cos}[3\,x]}{3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]}}\,\mathrm{Tan}\left[\frac{x}{2}\right]}{3-2\,\mathrm{Cos}[x]+\mathrm{Cos}[2\,x]}\right)\sqrt{\frac{\sqrt{3}-3\,\mathrm{i}\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\sqrt{\frac{\sqrt{3}+3\,\mathrm{i}\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{3\,\mathrm{i}+\sqrt{3}}}\right)\Big/$$

$$\left(\sqrt{3}\sqrt{\frac{\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3-\mathrm{i}\sqrt{3}}}\left(1+3\,\mathrm{Tan}\left[\frac{x}{2}\right]^4\right)\right)-\left(\left(2\,\mathrm{i}\sqrt{3}\,\mathrm{Cos}[x]^2\left(\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{3}\sqrt{\frac{\mathrm{i}\,\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\right],\frac{3\,\mathrm{i}-\sqrt{3}}{3\,\mathrm{i}+\sqrt{3}}\right]-\right.\right.\right.$$

$$\left.\mathrm{EllipticPi}\left[\frac{1}{6}\left(3+\mathrm{i}\sqrt{3}\right),\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{3}\sqrt{\frac{\mathrm{i}\,\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\right],\frac{3\,\mathrm{i}-\sqrt{3}}{3\,\mathrm{i}+\sqrt{3}}\right]\right)\mathrm{Sec}\left[\frac{x}{2}\right]^6\sqrt{\left(4+3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]\right)\mathrm{Sec}[x]^3}$$

$$\left.\mathrm{Tan}\left[\frac{x}{2}\right]^3\sqrt{\frac{\sqrt{3}-3\,\mathrm{i}\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\sqrt{\frac{\sqrt{3}+3\,\mathrm{i}\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{3\,\mathrm{i}+\sqrt{3}}}\right)\Big/\left(\sqrt{\frac{\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3-\mathrm{i}\sqrt{3}}}\left(1+3\,\mathrm{Tan}\left[\frac{x}{2}\right]^4\right)^2\right)+$$

$$\Bigg(\sqrt{3}\ \mathrm{Cos}[x]^2\ \Bigg(\mathrm{EllipticF}\Big[i\ \mathrm{ArcSinh}\Big[\sqrt{3}\ \sqrt{\frac{i\ \mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \Big],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\Big]-\mathrm{EllipticPi}\Big[\frac{1}{6}\left(3+i\,\sqrt{3}\right),$$

$$i\ \mathrm{ArcSinh}\Big[\sqrt{3}\ \sqrt{\frac{i\ \mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \Big],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\Big]\Bigg)\ \mathrm{Sec}\Big[\frac{x}{2}\Big]^6\ \sqrt{\left(4+3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]\right)\ \mathrm{Sec}[x]^3}$$

$$\mathrm{Tan}\Big[\frac{x}{2}\Big]\ \sqrt{\frac{\sqrt{3}-3\,i\ \mathrm{Tan}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\Bigg)\Bigg/\Bigg(2\,\left(3\,i+\sqrt{3}\right)\ \sqrt{\frac{\mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3-i\,\sqrt{3}}}\ \sqrt{\frac{\sqrt{3}+3\,i\ \mathrm{Tan}\big[\frac{x}{2}\big]^2}{3\,i+\sqrt{3}}}\ \left(1+3\,\mathrm{Tan}\Big[\frac{x}{2}\Big]^4\right)\Bigg)-$$

$$\Bigg(\sqrt{3}\ \mathrm{Cos}[x]^2\ \Bigg(\mathrm{EllipticF}\Big[i\ \mathrm{ArcSinh}\Big[\sqrt{3}\ \sqrt{\frac{i\ \mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \Big],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\Big]-\mathrm{EllipticPi}\Big[\frac{1}{6}\left(3+i\,\sqrt{3}\right),$$

$$i\ \mathrm{ArcSinh}\Big[\sqrt{3}\ \sqrt{\frac{i\ \mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \Big],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\Big]\Bigg)\ \mathrm{Sec}\Big[\frac{x}{2}\Big]^6\ \sqrt{\left(4+3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]\right)\ \mathrm{Sec}[x]^3}\ \mathrm{Tan}\Big[\frac{x}{2}\Big]\ \sqrt{\frac{\sqrt{3}+3\,i\ \mathrm{Tan}\big[\frac{x}{2}\big]^2}{3\,i+\sqrt{3}}}\Bigg)\Bigg/$$

$$\Bigg(2\,\left(-3\,i+\sqrt{3}\right)\ \sqrt{\frac{\mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3-i\,\sqrt{3}}}\ \sqrt{\frac{\sqrt{3}-3\,i\ \mathrm{Tan}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \left(1+3\,\mathrm{Tan}\Big[\frac{x}{2}\Big]^4\right)\Bigg)+\Bigg(2\,i\ \mathrm{Cos}[x]\ \Bigg(\mathrm{EllipticF}\Big[$$

$$i\ \mathrm{ArcSinh}\Big[\sqrt{3}\ \sqrt{\frac{i\ \mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \Big],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\Big]-\mathrm{EllipticPi}\Big[\frac{1}{6}\left(3+i\,\sqrt{3}\right),\ i\ \mathrm{ArcSinh}\Big[\sqrt{3}\ \sqrt{\frac{i\ \mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \Big],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\Big]\Bigg)$$

$$\mathrm{Sec}\Big[\frac{x}{2}\Big]^4\ \sqrt{\left(4+3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]\right)\ \mathrm{Sec}[x]^3}\ \mathrm{Sin}[x]\ \sqrt{\frac{\sqrt{3}-3\,i\ \mathrm{Tan}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \sqrt{\frac{\sqrt{3}+3\,i\ \mathrm{Tan}\big[\frac{x}{2}\big]^2}{3\,i+\sqrt{3}}}\Bigg)\Bigg/$$

$$\Bigg(\sqrt{3}\ \sqrt{\frac{\mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3-i\,\sqrt{3}}}\ \left(1+3\,\mathrm{Tan}\Big[\frac{x}{2}\Big]^4\right)\Bigg)-\Bigg(2\,i\ \mathrm{Cos}[x]^2\ \Bigg(\mathrm{EllipticF}\Big[i\ \mathrm{ArcSinh}\Big[\sqrt{3}\ \sqrt{\frac{i\ \mathrm{Cos}[x]\ \mathrm{Sec}\big[\frac{x}{2}\big]^2}{-3\,i+\sqrt{3}}}\ \Big],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\Big]-$$

$$\text{EllipticPi}\left[\frac{1}{6}\left(3+i\sqrt{3}\right),\ i\,\text{ArcSinh}\left[\sqrt{3}\sqrt{\frac{i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3\,i+\sqrt{3}}}\right],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\right]\text{Sec}\left[\frac{x}{2}\right]^4\sqrt{\left(4+3\,\text{Cos}[x]+\text{Cos}[3\,x]\right)\text{Sec}[x]^3}$$

$$\text{Tan}\left[\frac{x}{2}\right]\sqrt{\frac{\sqrt{3}-3\,i\,\text{Tan}\left[\frac{x}{2}\right]^2}{-3\,i+\sqrt{3}}}\sqrt{\frac{\sqrt{3}+3\,i\,\text{Tan}\left[\frac{x}{2}\right]^2}{3\,i+\sqrt{3}}}\Bigg)\Bigg/\left(\sqrt{3}\sqrt{\frac{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3-i\sqrt{3}}}\left(1+3\,\text{Tan}\left[\frac{x}{2}\right]^4\right)\right)+$$

$$\Bigg(i\,\text{Cos}[x]^2\Bigg(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{3}\sqrt{\frac{i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3\,i+\sqrt{3}}}\right],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\right]-\text{EllipticPi}\Bigg[\frac{1}{6}\left(3+i\sqrt{3}\right),$$

$$i\,\text{ArcSinh}\left[\sqrt{3}\sqrt{\frac{i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3\,i+\sqrt{3}}}\right],\ \frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\Bigg]\Bigg)\text{Sec}\left[\frac{x}{2}\right]^4\sqrt{\left(4+3\,\text{Cos}[x]+\text{Cos}[3\,x]\right)\text{Sec}[x]^3}\Bigg(-\frac{\text{Sec}\left[\frac{x}{2}\right]^2\text{Sin}[x]}{-3-i\sqrt{3}}+$$

$$\frac{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{-3-i\sqrt{3}}\Bigg)\sqrt{\frac{\sqrt{3}-3\,i\,\text{Tan}\left[\frac{x}{2}\right]^2}{-3\,i+\sqrt{3}}}\sqrt{\frac{\sqrt{3}+3\,i\,\text{Tan}\left[\frac{x}{2}\right]^2}{3\,i+\sqrt{3}}}\Bigg)\Bigg/\left(2\sqrt{3}\left(\frac{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3-i\sqrt{3}}\right)^{3/2}\left(1+3\,\text{Tan}\left[\frac{x}{2}\right]^4\right)\right)-$$

$$\Bigg(i\,\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4\sqrt{\left(4+3\,\text{Cos}[x]+\text{Cos}[3\,x]\right)\text{Sec}[x]^3}\sqrt{\frac{\sqrt{3}-3\,i\,\text{Tan}\left[\frac{x}{2}\right]^2}{-3\,i+\sqrt{3}}}\sqrt{\frac{\sqrt{3}+3\,i\,\text{Tan}\left[\frac{x}{2}\right]^2}{3\,i+\sqrt{3}}}$$

$$\Bigg(\frac{i\sqrt{3}\left(-\frac{i\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Sin}[x]}{-3\,i+\sqrt{3}}+\frac{i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{-3\,i+\sqrt{3}}\right)}{2\sqrt{\frac{i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3\,i+\sqrt{3}}}\sqrt{1+\frac{3\,i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3\,i+\sqrt{3}}}\sqrt{1+\frac{3\,i\left(3\,i-\sqrt{3}\right)\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{\left(-3\,i+\sqrt{3}\right)\left(3\,i+\sqrt{3}\right)}}}-\left(i\sqrt{3}\left(-\frac{i\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Sin}[x]}{-3\,i+\sqrt{3}}+\frac{i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{-3\,i+\sqrt{3}}\right)\right)\Bigg/$$

$$\left(2\sqrt{\frac{\mathrm{i}\,\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\sqrt{1+\frac{3\,\mathrm{i}\,\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\left(1+\frac{\mathrm{i}\left(3+\mathrm{i}\sqrt{3}\right)\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{2\left(-3\,\mathrm{i}+\sqrt{3}\right)}\right)\sqrt{1+\frac{3\,\mathrm{i}\left(3\,\mathrm{i}-\sqrt{3}\right)\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{\left(-3\,\mathrm{i}+\sqrt{3}\right)\left(3\,\mathrm{i}+\sqrt{3}\right)}}\right)\bigg)\bigg)\Big/$$

$$\left(\sqrt{3}\sqrt{\frac{\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3-\mathrm{i}\sqrt{3}}}\left(1+3\,\mathrm{Tan}\left[\frac{x}{2}\right]^4\right)\right)-\left(\mathrm{i}\,\mathrm{Cos}[x]^2\left(\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{3}\sqrt{\frac{\mathrm{i}\,\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\right],\frac{3\,\mathrm{i}-\sqrt{3}}{3\,\mathrm{i}+\sqrt{3}}\right]-\right.\right.$$

$$\left.\mathrm{EllipticPi}\left[\frac{1}{6}\left(3+\mathrm{i}\sqrt{3}\right),\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{3}\sqrt{\frac{\mathrm{i}\,\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}\right],\frac{3\,\mathrm{i}-\sqrt{3}}{3\,\mathrm{i}+\sqrt{3}}\right]\right)\mathrm{Sec}\left[\frac{x}{2}\right]^4\sqrt{\frac{\sqrt{3}-3\,\mathrm{i}\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{-3\,\mathrm{i}+\sqrt{3}}}$$

$$\left.\sqrt{\frac{\sqrt{3}+3\,\mathrm{i}\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{3\,\mathrm{i}+\sqrt{3}}}\left(\mathrm{Sec}[x]^3\left(-3\,\mathrm{Sin}[x]-3\,\mathrm{Sin}[3\,x]\right)+3\left(4+3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]\right)\mathrm{Sec}[x]^3\,\mathrm{Tan}[x]\right)\right)\Big/$$

$$\left.\left.\left.\left(2\sqrt{3}\sqrt{\frac{\mathrm{Cos}[x]\,\mathrm{Sec}\left[\frac{x}{2}\right]^2}{-3-\mathrm{i}\sqrt{3}}}\sqrt{\left(4+3\,\mathrm{Cos}[x]+\mathrm{Cos}[3\,x]\right)\mathrm{Sec}[x]^3}\left(1+3\,\mathrm{Tan}\left[\frac{x}{2}\right]^4\right)\right)\right)\right)\right)$$

Problem 8: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\sqrt{2+2\,\mathrm{Tan}[x]+\mathrm{Tan}[x]^2}\,dx$$

Optimal (type 3, 137 leaves, 9 steps) :

$$\text{ArcSinh}[1+\text{Tan}[x]]-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)\text{Tan}[x]}{\sqrt{10\left(1+\sqrt{5}\right)}\ \sqrt{2+2\,\text{Tan}[x]+\text{Tan}[x]^2}}\right]-$$

$$\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{2\sqrt{5}+\left(5-\sqrt{5}\right)\text{Tan}[x]}{\sqrt{10\left(-1+\sqrt{5}\right)}\ \sqrt{2+2\,\text{Tan}[x]+\text{Tan}[x]^2}}\right]$$

Result (type 4, 7376 leaves) :

$$-\frac{1}{(1+\text{Cos}[x])\sqrt{\frac{3+\text{Cos}[2x]+2\,\text{Sin}[2x]}{(1+\text{Cos}[x])^2}}}\,4\,\text{Cos}[x]$$

$$\Bigg(\Bigg(\Bigg(\text{EllipticF}\Bigg[\text{ArcSin}\Bigg[\sqrt{\Big(\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]+\text{Tan}[\tfrac{x}{2}]\big)\Big)\Big/}$$
$$\sqrt{\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]+\text{Tan}[\tfrac{x}{2}]\big)\Big)\Big)}\Bigg]\Bigg],$$
$$-\Big(\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,3]\big)\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]-$$
$$\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\Big)\Big/\Big(\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]+\text{Root}[1+2\#1-2\#1^3+\#1^4\&,3]\big)$$
$$\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\Big)\Big)\Bigg]\ \big(1-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]\big)-$$
$$\text{EllipticPi}\Big[\Big(\big(-1+\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]\big)\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]+\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\Big)\Big/$$
$$\Big(\big(-1+\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]\big)\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]+\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\Big),$$
$$\text{ArcSin}\Bigg[\sqrt{\Big(\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]+\text{Tan}[\tfrac{x}{2}]\big)\Big)\Big/}$$
$$\sqrt{\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]+\text{Tan}[\tfrac{x}{2}]\big)\Big)\Big)}\Bigg],$$
$$-\Big(\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,3]\big)\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]-$$
$$\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\Big)\Big/\Big(\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]+\text{Root}[1+2\#1-2\#1^3+\#1^4\&,3]\big)$$
$$\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\Big)\Big)\Big]\ \big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]+$$
$$\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]\big)\Bigg)\ \big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]+\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)$$

$$\sqrt{\Big(\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]+\text{Tan}[\tfrac{x}{2}]\big)\Big)\Big/}$$
$$\sqrt{\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\&,1]-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,4]\big)}$$
$$\sqrt{\big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]+\text{Tan}[\tfrac{x}{2}]\big)\Big)\Big)}\ \big(-\text{Root}[1+2\#1-2\#1^3+\#1^4\&,2]+\text{Tan}[\tfrac{x}{2}]\big)^2$$

```
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2]))/
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2]))/
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))])/
 ((-1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (1 - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
  Sqrt[1 + 2 Tan[x/2] - 2 Tan[x/2]^3 + Tan[x/2]^4]) -
((2 - ⅈ) (EllipticF[ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] +
          Tan[x/2]))/((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] +
          Tan[x/2])))]], -(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] -
         Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
        (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])))] (ⅈ - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) -
   EllipticPi[((-ⅈ + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/
     ((-ⅈ + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])),
    ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))/
       ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]],
    -(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] -
         Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
        (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])))] (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] +
     Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
  Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))/
    ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
```

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(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
 Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2]))/
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
 Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2]))/
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))])/
((-ⅈ + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (ⅈ - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
  Sqrt[1 + 2 Tan[x/2] - 2 Tan[x/2]^3 + Tan[x/2]^4]) -
((2 + ⅈ) (EllipticF[ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] +
          Tan[x/2]))/((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] +
          Tan[x/2])))]], -(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] -
         Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
        (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])))] (-ⅈ - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) -
    EllipticPi[((ⅈ + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/
      ((ⅈ + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])),
     ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))/
        ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]],
     -(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] -
          Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
         (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])))] (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] +
      Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
 Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))/
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((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
   (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2]))) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
 Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2]))/
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
 Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2]))/
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]) /
((I + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (-I - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
  Sqrt[1 + 2 Tan[x/2] - 2 Tan[x/2]^3 + Tan[x/2]^4]) -
((EllipticF[ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))/
      ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]],
    -(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] -
          Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
        (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])))] (-1 - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) -
   EllipticPi[((1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/
     ((1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])),
    ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))/
      ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]],
    -(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] -
          Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))/((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
        (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])))] (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] +
    Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
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Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
    ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))] (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2])) /
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2])) /
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]) /
((1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (-1 - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
  Sqrt[1 + 2 Tan[x/2] - 2 Tan[x/2]^3 + Tan[x/2]^4]) + (2 EllipticF[
  ArcSin[Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
      ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]],
  ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] -
       Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) / ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
     (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))]
  (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
  Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))] (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
  Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2])) /
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
```

$$\sqrt{\left(\left(\left(-\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,1\right]+\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,2\right]\right)\left(-\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,4\right]+\text{Tan}\left[\frac{x}{2}\right]\right)\right)\Big/\left(\left(-\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,1\right]+\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,4\right]\right)\left(-\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,2\right]+\text{Tan}\left[\frac{x}{2}\right]\right)\right)\right)}\Big/$$
$$\left(\left(-\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,1\right]+\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,2\right]\right)\left(-\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,2\right]+\text{Root}\left[1+2\,\#1-2\,\#1^3+\#1^4\,\&,\,4\right]\right)\right.$$
$$\left.\sqrt{1+2\,\text{Tan}\left[\frac{x}{2}\right]-2\,\text{Tan}\left[\frac{x}{2}\right]^3+\text{Tan}\left[\frac{x}{2}\right]^4}\right)\sqrt{2+2\,\text{Tan}[x]+\text{Tan}[x]^2}$$

Problem 9: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right]\text{Sin}[x]\,dx$$

Optimal (type 3, 41 leaves, 7 steps):

$$\frac{1}{2}\,\text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right]-\text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right]\text{Cos}[x]+\frac{1}{2}\,\text{Cos}[x]\sqrt{-1+\text{Sec}[x]}$$

Result (type 4, 285 leaves):

$$-\text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right]\text{Cos}[x]+\frac{1}{2}\,\text{Cos}[x]\sqrt{-1+\text{Sec}[x]}-\frac{1}{2}\left(-3-2\sqrt{2}\right)\text{Cos}\left[\frac{x}{4}\right]^2\left(1-\sqrt{2}+\left(-2+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]\right)$$
$$\text{Cot}\left[\frac{x}{4}\right]\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],\,17-12\sqrt{2}\right]+2\,\text{EllipticPi}\left[-3+2\sqrt{2},\,-\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],\,17-12\sqrt{2}\right]\right)$$
$$\sqrt{\left(7-5\sqrt{2}+\left(10-7\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]\right)\text{Sec}\left[\frac{x}{4}\right]^2}\;\sqrt{\left(-1-\sqrt{2}+\left(2+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]\right)\text{Sec}\left[\frac{x}{4}\right]^2}$$
$$\sqrt{-1+\text{Sec}[x]}\;\text{Sec}[x]\;\sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{x}{4}\right]^2}\;\sqrt{1+\left(-3+2\sqrt{2}\right)\text{Tan}\left[\frac{x}{4}\right]^2}$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{ArcTan}\left[x+\sqrt{1-x^2}\right]dx$$

Optimal (type 3, 141 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 1822 leaves):

$$x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{16}\left(-8\,\text{ArcSin}[x]+2\sqrt{2+2\,i\sqrt{3}}\,\text{ArcTan}\left[\left(\left(1+i\sqrt{3}-2x^2\right)\left(-1+x^2\right)\right)\Big/\left(-3\,i-\sqrt{3}+2\sqrt{3}\,x^4+x^3\left(-6-2\,i\sqrt{3}-2\sqrt{2-2\,i\sqrt{3}}\sqrt{1-x^2}\right)+x\left(6+2\,i\sqrt{3}-2\sqrt{2-2\,i\sqrt{3}}\sqrt{1-x^2}\right)+x^2\left(3\,i-\sqrt{3}+2\sqrt{6-6\,i\sqrt{3}}\sqrt{1-x^2}\right)\right)\right]-\right.$$

$$2\sqrt{2+2\,i\sqrt{3}}\,\text{ArcTan}\left[\left(\left(1+i\sqrt{3}-2x^2\right)\left(-1+x^2\right)\right)\Big/\left(-3\,i-\sqrt{3}+2\sqrt{3}\,x^4+2x\left(-3-i\sqrt{3}+\sqrt{2-2\,i\sqrt{3}}\sqrt{1-x^2}\right)+2x^3\left(3+i\sqrt{3}+\sqrt{2-2\,i\sqrt{3}}\sqrt{1-x^2}\right)+x^2\left(3\,i-\sqrt{3}+2\sqrt{6-6\,i\sqrt{3}}\sqrt{1-x^2}\right)\right)\right]-$$

$$2\sqrt{2-2\,i\sqrt{3}}\,\text{ArcTan}\left[\left(\left(-1+x^2\right)\left(-1+i\sqrt{3}+2x^2\right)\right)\Big/\left(3\,i-\sqrt{3}+2\sqrt{3}\,x^4+x\left(6-2\,i\sqrt{3}-2\sqrt{2+2\,i\sqrt{3}}\sqrt{1-x^2}\right)+x^3\left(-6+2\,i\sqrt{3}-2\sqrt{2+2\,i\sqrt{3}}\sqrt{1-x^2}\right)+x^2\left(-3\,i-\sqrt{3}+2\sqrt{6+6\,i\sqrt{3}}\sqrt{1-x^2}\right)\right)\right]+$$

$$2\sqrt{2-2\,i\sqrt{3}}\,\text{ArcTan}\left[\left(\left(-1+x^2\right)\left(-1+i\sqrt{3}+2x^2\right)\right)\Big/\left(3\,i-\sqrt{3}+2\sqrt{3}\,x^4+2x^3\left(3-i\sqrt{3}+\sqrt{2+2\,i\sqrt{3}}\sqrt{1-x^2}\right)+2x\left(-3+i\sqrt{3}+\sqrt{2+2\,i\sqrt{3}}\sqrt{1-x^2}\right)+x^2\left(-3\,i-\sqrt{3}+2\sqrt{6+6\,i\sqrt{3}}\sqrt{1-x^2}\right)\right)\right]-$$

$$2\,\text{Log}\left[-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^2\right]+2\,i\sqrt{3}\,\text{Log}\left[-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^2\right]-2\,\text{Log}\left[\frac{1}{2}\,i\left(i+\sqrt{3}\right)+x^2\right]-2\,i\sqrt{3}\,\text{Log}\left[\frac{1}{2}\,i\left(i+\sqrt{3}\right)+x^2\right]-$$

$$i\sqrt{2-2\,i\sqrt{3}}\,\text{Log}\left[16\left(1+\sqrt{3}\,x+x^2\right)^2\right]+i\sqrt{2+2\,i\sqrt{3}}\,\text{Log}\left[16\left(1+\sqrt{3}\,x+x^2\right)^2\right]+$$

$$i\sqrt{2-2\,i\sqrt{3}}\,\text{Log}\left[\left(4-4\sqrt{3}\,x+4x^2\right)^2\right]-i\sqrt{2+2\,i\sqrt{3}}\,\text{Log}\left[\left(4-4\sqrt{3}\,x+4x^2\right)^2\right]-$$

$$i\sqrt{2+2\,i\sqrt{3}}\,\text{Log}\left[3\,i+\sqrt{3}-\left(-i+\sqrt{3}\right)x^4+2\,i\sqrt{2-2\,i\sqrt{3}}\sqrt{1-x^2}+5\,i\,x^2\left(2+\sqrt{2-2\,i\sqrt{3}}\sqrt{1-x^2}\right)+x\left(3+5\,i\sqrt{3}+3\,i\sqrt{6-6\,i\sqrt{3}}\sqrt{1-x^2}\right)+i\,x^3\left(3\,i+3\sqrt{3}+\sqrt{6-6\,i\sqrt{3}}\sqrt{1-x^2}\right)\right]+$$

$$i\sqrt{2+2\,i\sqrt{3}}\,\text{Log}\left[3\,i+\sqrt{3}-\left(-i+\sqrt{3}\right)x^4+2\,i\sqrt{2-2\,i\sqrt{3}}\sqrt{1-x^2}+5\,i\,x^2\left(2+\sqrt{2-2\,i\sqrt{3}}\sqrt{1-x^2}\right)+x^3\left(3-3\,i\sqrt{3}-i\sqrt{6-6\,i\sqrt{3}}\sqrt{1-x^2}\right)-i\,x\left(-3\,i+5\sqrt{3}+3\sqrt{6-6\,i\sqrt{3}}\sqrt{1-x^2}\right)\right]+$$

$$i\sqrt{2-2i\sqrt{3}}\ \text{Log}\Big[-3i+\sqrt{3}-\left(i+\sqrt{3}\right)x^4-2i\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}-5ix^2\left(2+\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}\right)+$$
$$x\left(3-5i\sqrt{3}-3i\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}\right)-ix^3\left(-3i+3\sqrt{3}+\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}\right)\Big]-$$
$$i\sqrt{2-2i\sqrt{3}}\ \text{Log}\Big[-3i+\sqrt{3}-\left(i+\sqrt{3}\right)x^4-2i\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}-5ix^2\left(2+\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}\right)+$$
$$x^3\left(3+3i\sqrt{3}+i\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}\right)+ix\left(3i+5\sqrt{3}+3\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}\right)\Big]\Bigg)$$

Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]}{\sqrt{1-x^2}}\,dx$$

Optimal (type 3, 152 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\ \text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\ \text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-$$
$$\frac{1}{4}\sqrt{3}\ \text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]-\sqrt{1-x^2}\ \text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 2408 leaves) :

$$-\frac{\text{ArcSin}[x]}{2}-\sqrt{1-x^2}\ \text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{4\sqrt{6\left(1-i\sqrt{3}\right)}}$$
$$\left(-3i+\sqrt{3}\right)\text{ArcTan}\Big[\Big(3-i\sqrt{3}-12ix+4\sqrt{3}\,x-12i\sqrt{3}\,x^2-12ix^3-4\sqrt{3}\,x^3-3x^4-$$
$$i\sqrt{3}\,x^4-2i\sqrt{2\left(1-i\sqrt{3}\right)}\,x\sqrt{1-x^2}-2i\sqrt{6\left(1-i\sqrt{3}\right)}\,x^2\sqrt{1-x^2}-2i\sqrt{2\left(1-i\sqrt{3}\right)}\,x^3\sqrt{1-x^2}\Big)\Big/$$
$$\left(i-\sqrt{3}-6x+6i\sqrt{3}\,x+30ix^2-2\sqrt{3}\,x^2+6x^3+18i\sqrt{3}\,x^3+11ix^4+3\sqrt{3}\,x^4\right)\Big]-\frac{1}{4\sqrt{6\left(1-i\sqrt{3}\right)}}$$
$$\left(-3i+\sqrt{3}\right)\text{ArcTan}\Big[\Big(3-i\sqrt{3}+12ix-4\sqrt{3}\,x-12i\sqrt{3}\,x^2+12ix^3+4\sqrt{3}\,x^3-3x^4-i\sqrt{3}\,x^4+$$

$$2\,i\sqrt{2\left(1-i\sqrt{3}\right)}\;x\sqrt{1-x^2}-2\,i\sqrt{6\left(1-i\sqrt{3}\right)}\;x^2\sqrt{1-x^2}+2\,i\sqrt{2\left(1-i\sqrt{3}\right)}\;x^3\sqrt{1-x^2}\Big)\Big/$$

$$\left(i-\sqrt{3}+6\,x-6\,i\sqrt{3}\;x+30\,i\,x^2-2\sqrt{3}\;x^2-6\,x^3-18\,i\sqrt{3}\;x^3+11\,i\,x^4+3\sqrt{3}\;x^4\right)\Big]-\frac{1}{4\sqrt{6\left(1+i\sqrt{3}\right)}}$$

$$\left(3\,i+\sqrt{3}\right)\text{ArcTan}\Big[\Big(-3-i\sqrt{3}-12\,i\,x-4\sqrt{3}\;x-12\,i\sqrt{3}\;x^2-12\,i\,x^3+4\sqrt{3}\;x^3+3\,x^4-i\sqrt{3}\;x^4-$$

$$2\,i\sqrt{2\left(1+i\sqrt{3}\right)}\;x\sqrt{1-x^2}-2\,i\sqrt{6\left(1+i\sqrt{3}\right)}\;x^2\sqrt{1-x^2}-2\,i\sqrt{2\left(1+i\sqrt{3}\right)}\;x^3\sqrt{1-x^2}\Big)\Big/$$

$$\left(-i-\sqrt{3}-6\,x-6\,i\sqrt{3}\;x-30\,i\,x^2-2\sqrt{3}\;x^2+6\,x^3-18\,i\sqrt{3}\;x^3-11\,i\,x^4+3\sqrt{3}\;x^4\right)\Big]+\frac{1}{4\sqrt{6\left(1+i\sqrt{3}\right)}}$$

$$\left(3\,i+\sqrt{3}\right)\text{ArcTan}\Big[\Big(-3-i\sqrt{3}+12\,i\,x+4\sqrt{3}\;x-12\,i\sqrt{3}\;x^2+12\,i\,x^3-4\sqrt{3}\;x^3+3\,x^4-i\sqrt{3}\;x^4+$$

$$2\,i\sqrt{2\left(1+i\sqrt{3}\right)}\;x\sqrt{1-x^2}-2\,i\sqrt{6\left(1+i\sqrt{3}\right)}\;x^2\sqrt{1-x^2}+2\,i\sqrt{2\left(1+i\sqrt{3}\right)}\;x^3\sqrt{1-x^2}\Big)\Big/$$

$$\left(-i-\sqrt{3}+6\,x+6\,i\sqrt{3}\;x-30\,i\,x^2-2\sqrt{3}\;x^2-6\,x^3+18\,i\sqrt{3}\;x^3-11\,i\,x^4+3\sqrt{3}\;x^4\right)\Big]-$$

$$\frac{i\left(-3\,i+\sqrt{3}\right)\text{Log}\Big[\left(-i+\sqrt{3}-2\,x\right)^2\left(i+\sqrt{3}-2\,x\right)^2\Big]}{8\sqrt{6\left(1-i\sqrt{3}\right)}}+\frac{i\left(3\,i+\sqrt{3}\right)\text{Log}\Big[\left(-i+\sqrt{3}-2\,x\right)^2\left(i+\sqrt{3}-2\,x\right)^2\Big]}{8\sqrt{6\left(1+i\sqrt{3}\right)}}+$$

$$\frac{i\left(-3\,i+\sqrt{3}\right)\text{Log}\Big[\left(-i+\sqrt{3}+2\,x\right)^2\left(i+\sqrt{3}+2\,x\right)^2\Big]}{8\sqrt{6\left(1-i\sqrt{3}\right)}}-$$

$$\frac{i\left(3\,i+\sqrt{3}\right)\text{Log}\Big[\left(-i+\sqrt{3}+2\,x\right)^2\left(i+\sqrt{3}+2\,x\right)^2\Big]}{8\sqrt{6\left(1+i\sqrt{3}\right)}}+$$

$$\frac{\left(3\,i+\sqrt{3}\right)\text{Log}\Big[-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^2\Big]}{8\sqrt{3}}+$$

$$\frac{\left(-3\,i+\sqrt{3}\right)\,\text{Log}\left[-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^2\right]}{8\sqrt{3}}+$$

$$\frac{1}{8\sqrt{6\left(1-i\sqrt{3}\right)}}$$

$$i\left(-3\,i+\sqrt{3}\right)\text{Log}\Big[3\,i+\sqrt{3}-3\,x-5\,i\sqrt{3}\,x+10\,i\,x^2+3\,x^3-3\,i\sqrt{3}\,x^3+i\,x^4-\sqrt{3}\,x^4+2\,i\sqrt{2\left(1-i\sqrt{3}\right)}\,\sqrt{1-x^2}-$$
$$3\,i\sqrt{6\left(1-i\sqrt{3}\right)}\,x\sqrt{1-x^2}+5\,i\sqrt{2\left(1-i\sqrt{3}\right)}\,x^2\sqrt{1-x^2}-i\sqrt{6\left(1-i\sqrt{3}\right)}\,x^3\sqrt{1-x^2}\Big]-\frac{1}{8\sqrt{6\left(1-i\sqrt{3}\right)}}$$

$$i\left(-3\,i+\sqrt{3}\right)\text{Log}\Big[3\,i+\sqrt{3}+3\,x+5\,i\sqrt{3}\,x+10\,i\,x^2-3\,x^3+3\,i\sqrt{3}\,x^3+i\,x^4-\sqrt{3}\,x^4+2\,i\sqrt{2\left(1-i\sqrt{3}\right)}\,\sqrt{1-x^2}+$$
$$3\,i\sqrt{6\left(1-i\sqrt{3}\right)}\,x\sqrt{1-x^2}+5\,i\sqrt{2\left(1-i\sqrt{3}\right)}\,x^2\sqrt{1-x^2}+i\sqrt{6\left(1-i\sqrt{3}\right)}\,x^3\sqrt{1-x^2}\Big]+\frac{1}{8\sqrt{6\left(1+i\sqrt{3}\right)}}$$

$$i\left(3\,i+\sqrt{3}\right)\text{Log}\Big[-3\,i+\sqrt{3}+3\,x-5\,i\sqrt{3}\,x-10\,i\,x^2-3\,x^3-3\,i\sqrt{3}\,x^3-i\,x^4-\sqrt{3}\,x^4-2\,i\sqrt{2\left(1+i\sqrt{3}\right)}\,\sqrt{1-x^2}-$$
$$3\,i\sqrt{6\left(1+i\sqrt{3}\right)}\,x\sqrt{1-x^2}-5\,i\sqrt{2\left(1+i\sqrt{3}\right)}\,x^2\sqrt{1-x^2}-i\sqrt{6\left(1+i\sqrt{3}\right)}\,x^3\sqrt{1-x^2}\Big]-\frac{1}{8\sqrt{6\left(1+i\sqrt{3}\right)}}$$

$$i\left(3\,i+\sqrt{3}\right)\text{Log}\Big[-3\,i+\sqrt{3}-3\,x+5\,i\sqrt{3}\,x-10\,i\,x^2+3\,x^3+3\,i\sqrt{3}\,x^3-i\,x^4-\sqrt{3}\,x^4-2\,i\sqrt{2\left(1+i\sqrt{3}\right)}\,\sqrt{1-x^2}+$$
$$3\,i\sqrt{6\left(1+i\sqrt{3}\right)}\,x\sqrt{1-x^2}-5\,i\sqrt{2\left(1+i\sqrt{3}\right)}\,x^2\sqrt{1-x^2}+i\sqrt{6\left(1+i\sqrt{3}\right)}\,x^3\sqrt{1-x^2}\Big]$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Log}\left[x+\sqrt{-1+x^2}\right]}{\left(1+x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$-\frac{1}{2}\text{ArcCosh}\left[x^2\right]+\frac{x\,\text{Log}\left[x+\sqrt{-1+x^2}\right]}{\sqrt{1+x^2}}$$

Result (type 3, 89 leaves):

$$\frac{4\,x\,\text{Log}\left[x+\sqrt{-1+x^2}\right]+\frac{\sqrt{-1+x^2}\,\left(1+x^2\right)\left(\text{Log}\left[1-\frac{x^2}{\sqrt{-1+x^4}}\right]-\text{Log}\left[1+\frac{x^2}{\sqrt{-1+x^4}}\right]\right)}{\sqrt{-1+x^4}}}{4\sqrt{1+x^2}}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3\,\text{ArcSin}[x]}{\sqrt{1-x^4}}\,dx$$

Optimal (type 3, 38 leaves, 5 steps):

$$\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{2}\sqrt{1-x^4}\,\text{ArcSin}[x]+\frac{\text{ArcSinh}[x]}{4}$$

Result (type 3, 85 leaves):

$$\frac{1}{4}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1-x^2}}-2\sqrt{1-x^4}\,\text{ArcSin}[x]+\text{Log}\left[1-x^2\right]-\text{Log}\left[-x+x^3+\sqrt{1-x^2}\,\sqrt{1-x^4}\right]\right)$$

Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{Sin}[x]}{1+\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 46 leaves):

$$-\frac{i\left(\text{ArcTan}\left[\frac{-i+\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right]-\text{ArcTan}\left[\frac{i+\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right]\right)}{\sqrt{2}}$$

Problem 38: Result unnecessarily involves higher level functions.

$$\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{\sqrt{2}}$$

Result (type 4, 36 leaves):

$$(-1)^{1/4}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]-2\,\text{EllipticPi}\left[i,\text{ArcSin}\left[(-1)^{3/4}x\right],-1\right]\right)$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{\sqrt{2}}$$

Result (type 4, 40 leaves):

$$(-1)^{1/4}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]-2\,\text{EllipticPi}\left[-i,i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \text{Log}[\text{Sin}[x]]\sqrt{1+\text{Sin}[x]}\,dx$$

Optimal (type 3, 42 leaves, 6 steps):

$$-4\,\text{ArcTanh}\left[\frac{\text{Cos}[x]}{\sqrt{1+\text{Sin}[x]}}\right]+\frac{4\,\text{Cos}[x]}{\sqrt{1+\text{Sin}[x]}}-\frac{2\,\text{Cos}[x]\,\text{Log}[\text{Sin}[x]]}{\sqrt{1+\text{Sin}[x]}}$$

Result (type 3, 87 leaves):

$$\frac{1}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}$$

$$2\left(-\text{Log}\left[1+\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}\left[1-\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]-\text{Cos}\left[\frac{x}{2}\right]\left(-2+\text{Log}[\text{Sin}[x]]\right)+\left(-2+\text{Log}[\text{Sin}[x]]\right)\text{Sin}\left[\frac{x}{2}\right]\right)\sqrt{1+\text{Sin}[x]}$$

Problem 44: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[x]}{\sqrt{1-\text{Sin}[x]^6}}\,dx$$

Optimal (type 3, 39 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,\text{Cos}[x]\left(1+\text{Sin}[x]^2\right)}{2\sqrt{1-\text{Sin}[x]^6}}\right]}{2\sqrt{3}}$$

Result (type 4, 5825 leaves):

$$-\Bigg(\Bigg((-1)^{3/4}\Bigg(\left(3\,i+(1+2\,i)\sqrt{2}\,3^{1/4}+(1+2\,i)\sqrt{3}+i\sqrt{2}\,3^{3/4}\right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2\,i+2\,(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}\right],8-4\sqrt{3}\right]-2\times 3^{1/4}\left(\sqrt{2}+3^{1/4}\right)\text{EllipticPi}\Bigg[$$

$$\frac{6\,(-3)^{1/4}-2\,(-3)^{3/4}+4\sqrt{3}}{3+3\sqrt{2}\,3^{1/4}+(2-i)\sqrt{3}+\sqrt{2}\,3^{3/4}},\text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2\,i+2\,(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}\right],8-4\sqrt{3}\Bigg]\Bigg)$$

$$\text{Sin}[x]\sqrt{\frac{2\,i-2\,(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}{\left(-i\sqrt{2}+3^{1/4}\right)\left(2\,i+2\,(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)}}\left(2-2\,(-1)^{3/4}\,3^{1/4}-i\sqrt{3}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\sqrt{-\frac{\left(i\sqrt{2}+3^{1/4}\right)\left(-i+2\left(-2\,i+\sqrt{3}\right)\text{Tan}\left[\frac{x}{2}\right]^2-i\,\text{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2\,i+2\,(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\Bigg)\Bigg/$$

$$\Big(\sqrt{2}\; 3^{1/4} \left(\left(3+6\,i\right)\sqrt{2}+\left(6+6\,i\right)3^{1/4}+\left(2+2\,i\right)3^{3/4}+\left(3+2\,i\right)\sqrt{6}\right)\sqrt{1-\mathrm{Sin}[x]^6}\;\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\sqrt{\frac{1+8\,\mathrm{Tan}\left[\frac{x}{2}\right]^2+30\,\mathrm{Tan}\left[\frac{x}{2}\right]^4+8\,\mathrm{Tan}\left[\frac{x}{2}\right]^6+\mathrm{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^4}}\;\Big(-\Big(\Big((-1)^{3/4}\sqrt{2}\;\Big(\left(3\,i+\left(1+2\,i\right)\sqrt{2}\;3^{1/4}+\left(1+2\,i\right)\sqrt{3}+i\,\sqrt{2}\;3^{3/4}\right)\mathrm{EllipticF}\Big[$$

$$\mathrm{ArcSin}\left[\frac{1}{2}\sqrt{\frac{\left(1+i\right)\left(\left(2+\sqrt{2}\;3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\,\sqrt{2}\;3^{1/4}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}}\right],\,8-4\sqrt{3}\Big]-2\times 3^{1/4}\left(\sqrt{2}+3^{1/4}\right)\mathrm{EllipticPi}\Big[$$

$$\frac{6\left(-3\right)^{1/4}-2\left(-3\right)^{3/4}+4\sqrt{3}}{3+3\sqrt{2}\;3^{1/4}+\left(2-i\right)\sqrt{3}+\sqrt{2}\;3^{3/4}},\,\mathrm{ArcSin}\left[\frac{1}{2}\sqrt{\frac{\left(1+i\right)\left(\left(2+\sqrt{2}\;3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\,\sqrt{2}\;3^{1/4}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}}\right],\,8-4\sqrt{3}\Big]\Big)$$

$$\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]\sqrt{\frac{2\,i-2\left(-3\right)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{\left(-i\,\sqrt{2}+3^{1/4}\right)\left(2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}}\;\left(2-2\left(-1\right)^{3/4}3^{1/4}-i\,\sqrt{3}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)$$

$$\sqrt{-\frac{\left(i\,\sqrt{2}+3^{1/4}\right)\left(-i+2\left(-2\,i+\sqrt{3}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2-i\,\mathrm{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\Big)\Big/\Big(3^{1/4}\left(\left(3+6\,i\right)\sqrt{2}+\left(6+6\,i\right)3^{1/4}+\left(2+2\,i\right)3^{3/4}+\left(3+2\,i\right)\sqrt{6}\right)$$

$$\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2\sqrt{\frac{1+8\,\mathrm{Tan}\left[\frac{x}{2}\right]^2+30\,\mathrm{Tan}\left[\frac{x}{2}\right]^4+8\,\mathrm{Tan}\left[\frac{x}{2}\right]^6+\mathrm{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^4}}\Big)\Big)+$$

$$\Big((-1)^{3/4}\sqrt{2}\;\Big(\left(3\,i+\left(1+2\,i\right)\sqrt{2}\;3^{1/4}+\left(1+2\,i\right)\sqrt{3}+i\,\sqrt{2}\;3^{3/4}\right)\mathrm{EllipticF}\Big[\mathrm{ArcSin}\Big[\frac{1}{2}$$

$$\sqrt{\frac{\left(1+i\right)\left(\left(2+\sqrt{2}\;3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\,\sqrt{2}\;3^{1/4}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}}\Big],\,8-4\sqrt{3}\Big]-2\times 3^{1/4}\left(\sqrt{2}+3^{1/4}\right)\mathrm{EllipticPi}\Big[$$

$$\frac{6(-3)^{1/4}-2(-3)^{3/4}+4\sqrt{3}}{3+3\sqrt{2}\,3^{1/4}+(2-i)\sqrt{3}+\sqrt{2}\,3^{3/4}},\ \text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}\right],\ 8-4\sqrt{3}\Bigg]\Bigg)$$

$$\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]\sqrt{\frac{2i-2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}{\left(-i\sqrt{2}+3^{1/4}\right)\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)}}\left(2-2(-1)^{3/4}3^{1/4}-i\sqrt{3}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\sqrt{-\frac{\left(i\sqrt{2}+3^{1/4}\right)\left(-i+2\left(-2i+\sqrt{3}\right)\text{Tan}\left[\frac{x}{2}\right]^2-i\,\text{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\Bigg)\Bigg/$$

$$\Bigg(3^{1/4}\left(\left(3+6i\right)\sqrt{2}+(6+6i)\,3^{1/4}+(2+2i)\,3^{3/4}+(3+2i)\sqrt{6}\right)\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^3\sqrt{\frac{1+8\,\text{Tan}\left[\frac{x}{2}\right]^2+30\,\text{Tan}\left[\frac{x}{2}\right]^4+8\,\text{Tan}\left[\frac{x}{2}\right]^6+\text{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}}\Bigg)-$$

$$\Bigg((-1)^{3/4}\Bigg(\left(3i+(1+2i)\sqrt{2}\,3^{1/4}+(1+2i)\sqrt{3}+i\sqrt{2}\,3^{3/4}\right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}\right],\ 8-4\sqrt{3}\right]-2\times3^{1/4}\left(\sqrt{2}+3^{1/4}\right)\text{EllipticPi}\Bigg[$$

$$\frac{6(-3)^{1/4}-2(-3)^{3/4}+4\sqrt{3}}{3+3\sqrt{2}\,3^{1/4}+(2-i)\sqrt{3}+\sqrt{2}\,3^{3/4}},\ \text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}\right],\ 8-4\sqrt{3}\Bigg]\Bigg)$$

$$\left(2-2(-1)^{3/4}3^{1/4}-i\sqrt{3}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2\sqrt{-\frac{\left(i\sqrt{2}+3^{1/4}\right)\left(-i+2\left(-2i+\sqrt{3}\right)\text{Tan}\left[\frac{x}{2}\right]^2-i\,\text{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}$$

$$\left(-\frac{i\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]\left(2i-2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\left(-i\sqrt{2}+3^{1/4}\right)\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{i\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{\left(-i\sqrt{2}+3^{1/4}\right)\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)}\right)\Bigg)\Bigg/$$

$$\Bigg(2\sqrt{2}\,3^{1/4}\left(\left(3+6\,i\right)\sqrt{2}+\left(6+6\,i\right)3^{1/4}+\left(2+2\,i\right)3^{3/4}+\left(3+2\,i\right)\sqrt{6}\right)\sqrt{\frac{2\,i-2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}{\left(-i\sqrt{2}+3^{1/4}\right)\left(2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)}}$$

$$\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2\sqrt{\frac{1+8\,\text{Tan}\left[\frac{x}{2}\right]^2+30\,\text{Tan}\left[\frac{x}{2}\right]^4+8\,\text{Tan}\left[\frac{x}{2}\right]^6+\text{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}}\Bigg)-\Bigg(\left(-1\right)^{3/4}\Bigg(\left(3\,i+\left(1+2\,i\right)\sqrt{2}\,3^{1/4}+\left(1+2\,i\right)\sqrt{3}+i\sqrt{2}\,3^{3/4}\right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{\left(1+i\right)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}\right],8-4\sqrt{3}\right]-2\times 3^{1/4}\left(\sqrt{2}+3^{1/4}\right)\text{EllipticPi}\Big[$$

$$\frac{6\left(-3\right)^{1/4}-2\left(-3\right)^{3/4}+4\sqrt{3}}{3+3\sqrt{2}\,3^{1/4}+\left(2-i\right)\sqrt{3}+\sqrt{2}\,3^{3/4}},\text{ArcSin}\left[\frac{1}{2}\sqrt{\frac{\left(1+i\right)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}\right],8-4\sqrt{3}\Big]\Bigg)$$

$$\sqrt{\frac{2\,i-2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}{\left(-i\sqrt{2}+3^{1/4}\right)\left(2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)}}\left(2-2\left(-1\right)^{3/4}3^{1/4}-i\sqrt{3}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\Bigg(-\frac{\left(i\sqrt{2}+3^{1/4}\right)\left(2\left(-2\,i+\sqrt{3}\right)\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]-2\,i\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]^3\right)}{\left(2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+$$

$$\frac{2\,i\left(i\sqrt{2}+3^{1/4}\right)\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]\left(-i+2\left(-2\,i+\sqrt{3}\right)\text{Tan}\left[\frac{x}{2}\right]^2-i\,\text{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}\Bigg)\Bigg)\Bigg/$$

$$\Bigg(2\sqrt{2}\,3^{1/4}\left(\left(3+6\,i\right)\sqrt{2}+\left(6+6\,i\right)3^{1/4}+\left(2+2\,i\right)3^{3/4}+\left(3+2\,i\right)\sqrt{6}\right)\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\sqrt{-\frac{\left(i\sqrt{2}+3^{1/4}\right)\left(-i+2\left(-2\,i+\sqrt{3}\right)\text{Tan}\left[\frac{x}{2}\right]^2-i\,\text{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2\,i+2\left(-3\right)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\sqrt{\frac{1+8\,\text{Tan}\left[\frac{x}{2}\right]^2+30\,\text{Tan}\left[\frac{x}{2}\right]^4+8\,\text{Tan}\left[\frac{x}{2}\right]^6+\text{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}}\Bigg)+$$

$$\Bigg(\left(-1\right)^{3/4}\Bigg(\left(3\,i+\left(1+2\,i\right)\sqrt{2}\,3^{1/4}+\left(1+2\,i\right)\sqrt{3}+i\sqrt{2}\,3^{3/4}\right)\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{1}{2}$$

$$\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}}\Big],8-4\sqrt{3}\Big]-2\times 3^{1/4}\left(\sqrt{2}+3^{1/4}\right)\mathrm{EllipticPi}\Big[$$

$$\frac{6(-3)^{1/4}-2(-3)^{3/4}+4\sqrt{3}}{3+3\sqrt{2}\,3^{1/4}+(2-i)\sqrt{3}+\sqrt{2}\,3^{3/4}},\mathrm{ArcSin}\Big[\frac{1}{2}\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}}\Big],8-4\sqrt{3}\Big]\Bigg)$$

$$\sqrt{\frac{2i-2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{\left(-i\sqrt{2}+3^{1/4}\right)\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}}\left(2-2(-1)^{3/4}3^{1/4}-i\sqrt{3}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\sqrt{-\frac{\left(i\sqrt{2}+3^{1/4}\right)\left(-i+2\left(-2i+\sqrt{3}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2-i\,\mathrm{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2}}$$

$$\Bigg(\frac{8\,\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]+60\,\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]^3+24\,\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]^5+4\,\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]^7}{\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^4}-$$

$$\frac{4\,\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]\left(1+8\,\mathrm{Tan}\left[\frac{x}{2}\right]^2+30\,\mathrm{Tan}\left[\frac{x}{2}\right]^4+8\,\mathrm{Tan}\left[\frac{x}{2}\right]^6+\mathrm{Tan}\left[\frac{x}{2}\right]^8\right)}{\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^5}\Bigg)\Bigg)\Big/\Bigg(2\sqrt{2}\,3^{1/4}$$

$$\left(\left(3+6i\right)\sqrt{2}+(6+6i)\,3^{1/4}+(2+2i)\,3^{3/4}+(3+2i)\sqrt{6}\right)\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2\left(\frac{1+8\,\mathrm{Tan}\left[\frac{x}{2}\right]^2+30\,\mathrm{Tan}\left[\frac{x}{2}\right]^4+8\,\mathrm{Tan}\left[\frac{x}{2}\right]^6+\mathrm{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^4}\right)^{3/2}\Bigg)-$$

$$\Bigg((-1)^{3/4}\sqrt{\frac{2i-2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{\left(-i\sqrt{2}+3^{1/4}\right)\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}}\left(2-2(-1)^{3/4}3^{1/4}-i\sqrt{3}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\sqrt{-\frac{\left(i\sqrt{2}+3^{1/4}\right)\left(-i+2\left(-2i+\sqrt{3}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2-i\,\mathrm{Tan}\left[\frac{x}{2}\right]^4\right)}{\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\Bigg(\Bigg(\left(3i+(1+2i)\sqrt{2}\,3^{1/4}+(1+2i)\sqrt{3}+i\sqrt{2}\,3^{3/4}\right)$$

$$\left(\frac{(1+i)\left(2-i\sqrt{2}\,3^{1/4}\right)\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}+\frac{(1-i)\,\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2}\right)\Bigg)\Big/$$

$$\Bigg(4\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}$$

$$\sqrt{1-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}$$

$$\sqrt{1-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(8-4\sqrt{3}\right)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}\Bigg)-\Bigg(3^{1/4}\left(\sqrt{2}+3^{1/4}\right)$$

$$\left(\frac{(1+i)\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}+\frac{(1-i)\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}\right)\Bigg)\Big/$$

$$\Bigg(2\sqrt{\frac{(1+i)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}$$

$$\sqrt{1-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}$$

$$\sqrt{1-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(8-4\sqrt{3}\right)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)}{2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2}}$$

$$\Bigg(1-\Bigg(\left(\frac{1}{4}+\frac{i}{4}\right)\left(6(-3)^{1/4}-2(-3)^{3/4}+4\sqrt{3}\right)\left(\left(2+\sqrt{2}\,3^{1/4}\right)\left(2+\sqrt{3}\right)+\left(2-i\sqrt{2}\,3^{1/4}\right)\text{Tan}\left[\frac{x}{2}\right]^2\right)\Bigg)\Big/$$

$$\left(\left(3+3\sqrt{2}\,3^{1/4}+(2-i)\sqrt{3}+\sqrt{2}\,3^{3/4}\right)\left(2i+2(-3)^{1/4}+\sqrt{3}+i\,\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Bigg)\Bigg)\Big/\Bigg(\sqrt{2}\,3^{1/4}$$

$$\left((3+6i)\sqrt{2}+(6+6i)\,3^{1/4}+(2+2i)\,3^{3/4}+(3+2i)\sqrt{6}\right)\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2\sqrt{\frac{1+8\,\text{Tan}\left[\frac{x}{2}\right]^2+30\,\text{Tan}\left[\frac{x}{2}\right]^4+8\,\text{Tan}\left[\frac{x}{2}\right]^6+\text{Tan}\left[\frac{x}{2}\right]^8}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}}\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{ArcTan}\left[x\sqrt{1+x^2}\right]dx$$

Optimal (type 3, 120 leaves, 12 steps):

$$x\,\text{ArcTan}\left[x\sqrt{1+x^2}\right]+\frac{1}{2}\text{ArcTan}\left[\sqrt{3}-2\sqrt{1+x^2}\right]-\frac{1}{2}\text{ArcTan}\left[\sqrt{3}+2\sqrt{1+x^2}\right]-\frac{1}{4}\sqrt{3}\,\text{Log}\left[2+x^2-\sqrt{3}\sqrt{1+x^2}\right]+\frac{1}{4}\sqrt{3}\,\text{Log}\left[2+x^2+\sqrt{3}\sqrt{1+x^2}\right]$$

Result (type 3, 116 leaves):

$$\frac{1}{2}\left(-\sqrt{-2+2\,i\sqrt{3}}\,\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{1+x^2}}{\sqrt{-1-i\sqrt{3}}}\right]-\sqrt{-2-2\,i\sqrt{3}}\,\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{1+x^2}}{\sqrt{-1+i\sqrt{3}}}\right]+2\,x\,\text{ArcTan}\left[x\sqrt{1+x^2}\right]\right)$$

Test results for the 284 problems in "Hearn Problems.m"

Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+x^2+x^4}dx$$

Optimal (type 3, 67 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{4}\text{Log}\left[1-x+x^2\right]+\frac{1}{4}\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 73 leaves):

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\,\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]-\sqrt{1+i\sqrt{3}}\,\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]\right)}{\sqrt{6}}$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{2+x^2+x^4}dx$$

Optimal (type 3, 196 leaves, 9 steps):

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right]+$$
$$\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right]-\frac{\text{Log}\left[\sqrt{2}-\sqrt{-1+2\sqrt{2}}\ x+x^2\right]}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\text{Log}\left[\sqrt{2}+\sqrt{-1+2\sqrt{2}}\ x+x^2\right]}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

Result (type 3, 91 leaves):

$$-\frac{i\,\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right]}{\sqrt{\frac{7}{2}\left(1-i\sqrt{7}\right)}}+\frac{i\,\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right]}{\sqrt{\frac{7}{2}\left(1+i\sqrt{7}\right)}}$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{2-x^2+x^4}\,dx$$

Optimal (type 3, 196 leaves, 9 steps):

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{-1+2\sqrt{2}}}\right]+$$
$$\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{-1+2\sqrt{2}}}\right]-\frac{\text{Log}\left[\sqrt{2}-\sqrt{1+2\sqrt{2}}\ x+x^2\right]}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\text{Log}\left[\sqrt{2}+\sqrt{1+2\sqrt{2}}\ x+x^2\right]}{4\sqrt{2\left(1+2\sqrt{2}\right)}}$$

Result (type 3, 91 leaves):

$$-\frac{i\,\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{7}\right)}}\right]}{\sqrt{\frac{7}{2}\left(-1-i\sqrt{7}\right)}}+\frac{i\,\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{7}\right)}}\right]}{\sqrt{\frac{7}{2}\left(-1+i\sqrt{7}\right)}}$$

Problem 51: Result is not expressed in closed-form.

$$\int \frac{1}{1 - x^4 + x^8}\, dx$$

Optimal (type 3, 275 leaves, 19 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right]}{2\sqrt{6}} - \frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right]}{2\sqrt{6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right]}{2\sqrt{6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right]}{2\sqrt{6}} -$$

$$\frac{\text{Log}\left[1-\sqrt{2-\sqrt{3}}\, x+x^2\right]}{4\sqrt{6}} + \frac{\text{Log}\left[1+\sqrt{2-\sqrt{3}}\, x+x^2\right]}{4\sqrt{6}} - \frac{\text{Log}\left[1-\sqrt{2+\sqrt{3}}\, x+x^2\right]}{4\sqrt{6}} + \frac{\text{Log}\left[1+\sqrt{2+\sqrt{3}}\, x+x^2\right]}{4\sqrt{6}}$$

Result (type 7, 42 leaves):

$$\frac{1}{4}\,\text{RootSum}\left[1 - \#1^4 + \#1^8\,\&,\ \frac{\text{Log}[x - \#1]}{-\#1^3 + 2\,\#1^7}\,\&\right]$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int \frac{x^7}{1 + x^{12}}\, dx$$

Optimal (type 3, 49 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2x^4}{\sqrt{3}}\right]}{4\sqrt{3}} - \frac{1}{12}\,\text{Log}\left[1+x^4\right] + \frac{1}{24}\,\text{Log}\left[1-x^4+x^8\right]$$

Result (type 3, 260 leaves):

$$\frac{1}{24}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}-2\sqrt{2}\,x}{1-\sqrt{3}}\right] - 2\sqrt{3}\,\text{ArcTan}\left[\frac{1-\sqrt{3}+2\sqrt{2}\,x}{1+\sqrt{3}}\right] + \right.$$

$$2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}+2\sqrt{2}\,x}{1+\sqrt{3}}\right] - 2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}+2\sqrt{2}\,x}{-1+\sqrt{3}}\right] - 2\,\text{Log}\left[1-\sqrt{2}\,x+x^2\right] - 2\,\text{Log}\left[1+\sqrt{2}\,x+x^2\right] +$$

$$\left.\text{Log}\left[2+\sqrt{2}\,x-\sqrt{6}\,x+2x^2\right] + \text{Log}\left[2+\sqrt{2}\,\left(-1+\sqrt{3}\right)x+2x^2\right] + \text{Log}\left[2-\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right] + \text{Log}\left[2+\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right]\right)$$

Problem 81: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x]\, dx$$

Optimal (type 3, 3 leaves, 1 step):

ArcTanh[Sin[x]]

Result (type 3, 33 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[x]\, dx$$

Optimal (type 3, 5 leaves, 1 step):

-ArcTanh[Cos[x]]

Result (type 3, 17 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[a+b\,x]\, dx$$

Optimal (type 3, 10 leaves, 1 step):

$$\frac{\text{Sin}[a+b\,x]}{b}$$

Result (type 3, 21 leaves):

$$\frac{\text{Cos}[b\,x]\,\text{Sin}[a]}{b}+\frac{\text{Cos}[a]\,\text{Sin}[b\,x]}{b}$$

Problem 111: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[a+b\,x]\, dx$$

Optimal (type 3, 12 leaves, 1 step) :

$$-\frac{\text{ArcTanh}[\text{Cos}[a+b\,x]]}{b}$$

Result (type 3, 38 leaves) :

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2}+\frac{b\,x}{2}\right]\right]}{b}+\frac{\text{Log}\left[\text{Sin}\left[\frac{a}{2}+\frac{b\,x}{2}\right]\right]}{b}$$

Problem 112: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[a+b\,x]\,dx$$

Optimal (type 3, 11 leaves, 1 step) :

$$\frac{\text{ArcTanh}[\text{Sin}[a+b\,x]]}{b}$$

Result (type 3, 68 leaves) :

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2}+\frac{b\,x}{2}\right]-\text{Sin}\left[\frac{a}{2}+\frac{b\,x}{2}\right]\right]}{b}+\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2}+\frac{b\,x}{2}\right]+\text{Sin}\left[\frac{a}{2}+\frac{b\,x}{2}\right]\right]}{b}$$

Problem 120: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1+\text{Sin}[x]}\,dx$$

Optimal (type 3, 10 leaves, 1 step) :

$$-\frac{\text{Cos}[x]}{1+\text{Sin}[x]}$$

Result (type 3, 23 leaves) :

$$\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}$$

Problem 121: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1-\text{Sin}[x]}\,dx$$

Optimal (type 3, 11 leaves, 1 step):

$$\frac{\text{Cos}[x]}{1-\text{Sin}[x]}$$

Result (type 3, 25 leaves):

$$\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}$$

Problem 190: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{-1+x^2}}\,dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Result (type 3, 38 leaves):

$$-\frac{1}{2}\,\text{Log}\left[1-\frac{x}{\sqrt{-1+x^2}}\right]+\frac{1}{2}\,\text{Log}\left[1+\frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 197: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x\sqrt{-1+x^2-x^4}}\,dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$-\frac{1}{2}\,\text{ArcTan}\left[\frac{2-x^2}{2\sqrt{-1+x^2-x^4}}\right]$$

Result (type 3, 37 leaves):

$$-i\,\text{Log}[x]+\frac{1}{2}\,i\,\text{Log}\left[-2+x^2+2\,i\sqrt{-1+x^2-x^4}\right]$$

Problem 202: Result more than twice size of optimal antiderivative.

$$\int\left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right)dx$$

Optimal (type 3, 27 leaves, 5 steps):

$$10\,\text{ArcTanh}\left[\frac{x}{\sqrt{-4+x^2}}\right]+\text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Result (type 3, 71 leaves):

$$-5\,\text{Log}\left[1-\frac{x}{\sqrt{-4+x^2}}\right]+5\,\text{Log}\left[1+\frac{x}{\sqrt{-4+x^2}}\right]-\frac{1}{2}\,\text{Log}\left[1-\frac{x}{\sqrt{-1+x^2}}\right]+\frac{1}{2}\,\text{Log}\left[1+\frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r\,\sqrt{-\text{alpha}^2-\text{epsilon}^2+2\,h\,r^2}}\,dr$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{-\text{alpha}^2-\text{epsilon}^2+2\,h\,r^2}}{\sqrt{\text{alpha}^2+\text{epsilon}^2}}\right]}{\sqrt{\text{alpha}^2+\text{epsilon}^2}}$$

Result (type 3, 58 leaves):

$$-\frac{i\,\text{Log}\left[\frac{2\left(-i\,\sqrt{\text{alpha}^2+\text{epsilon}^2}+\sqrt{-\text{alpha}^2-\text{epsilon}^2+2\,h\,r^2}\right)}{r}\right]}{\sqrt{\text{alpha}^2+\text{epsilon}^2}}$$

Problem 207: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r\,\sqrt{-\text{alpha}^2-2\,k\,r+2\,h\,r^2}}\,dr$$

Optimal (type 3, 37 leaves, 2 steps):

$$-\frac{\text{ArcTan}\left[\frac{\text{alpha}^2+k\,r}{\text{alpha}\,\sqrt{-\text{alpha}^2-2\,k\,r+2\,h\,r^2}}\right]}{\text{alpha}}$$

Result (type 3, 48 leaves):

$$-\frac{i \operatorname{Log}\left[\frac{2\left(-\frac{i\,(alpha^2+k\,r)}{alpha}+\sqrt{-alpha^2+2\,r\,(-k+h\,r)}\right)}{r}\right]}{alpha}$$

Problem 208: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r\,\sqrt{-alpha^2-epsilon^2-2\,k\,r+2\,h\,r^2}}\,dr$$

Optimal (type 3, 61 leaves, 2 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{alpha^2+epsilon^2+k\,r}{\sqrt{alpha^2+epsilon^2}\,\sqrt{-alpha^2-epsilon^2-2\,k\,r+2\,h\,r^2}}\right]}{\sqrt{alpha^2+epsilon^2}}$$

Result (type 3, 72 leaves):

$$-\frac{i \operatorname{Log}\left[\frac{2\left(-\frac{i\,(alpha^2+epsilon^2+k\,r)}{\sqrt{alpha^2+epsilon^2}}+\sqrt{-alpha^2-epsilon^2+2\,r\,(-k+h\,r)}\right)}{r}\right]}{\sqrt{alpha^2+epsilon^2}}$$

Problem 211: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{r}{\sqrt{-alpha^2+2\,e\,r^2-2\,k\,r^4}}\,dr$$

Optimal (type 3, 56 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{e-2\,k\,r^2}{\sqrt{2}\,\sqrt{k}\,\sqrt{-alpha^2+2\,e\,r^2-2\,k\,r^4}}\right]}{2\,\sqrt{2}\,\sqrt{k}}$$

Result (type 3, 66 leaves):

$$\frac{i \operatorname{Log}\left[-\frac{i\,\sqrt{2}\,(-e+2\,k\,r^2)}{\sqrt{k}}+2\,\sqrt{-alpha^2+2\,e\,r^2-2\,k\,r^4}\right]}{2\,\sqrt{2}\,\sqrt{k}}$$

Problem 213: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r\sqrt{-\text{alpha}^2+2\,h\,r^2-2\,k\,r^4}}\,dr$$

Optimal (type 3, 44 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{\text{alpha}^2-h\,r^2}{\text{alpha}\sqrt{-\text{alpha}^2+2\,h\,r^2-2\,k\,r^4}}\right]}{2\,\text{alpha}}$$

Result (type 3, 59 leaves):

$$-\frac{i\,\text{Log}\left[\frac{-2\,i\,\text{alpha}^2+2\,i\,h\,r^2+2\,\text{alpha}\sqrt{-\text{alpha}^2+2\,r^2\,(h-k\,r^2)}}{\text{alpha}\,r^2}\right]}{2\,\text{alpha}}$$

Problem 214: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r\sqrt{-\text{alpha}^2-\text{epsilon}^2+2\,h\,r^2-2\,k\,r^4}}\,dr$$

Optimal (type 3, 68 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{\text{alpha}^2+\text{epsilon}^2-h\,r^2}{\sqrt{\text{alpha}^2+\text{epsilon}^2}\sqrt{-\text{alpha}^2-\text{epsilon}^2+2\,h\,r^2-2\,k\,r^4}}\right]}{2\sqrt{\text{alpha}^2+\text{epsilon}^2}}$$

Result (type 3, 80 leaves):

$$-\frac{i\,\text{Log}\left[\frac{2\left(-\frac{i\,(\text{alpha}^2+\text{epsilon}^2-h\,r^2)}{\sqrt{\text{alpha}^2+\text{epsilon}^2}}+\sqrt{-\text{alpha}^2-\text{epsilon}^2+2\,r^2\,(h-k\,r^2)}\right)}{r^2}\right]}{2\sqrt{\text{alpha}^2+\text{epsilon}^2}}$$

Problem 235: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1+\text{Sin}[x]}\,dx$$

Optimal (type 3, 12 leaves, 1 step):

$$-\frac{2\,\text{Cos}[x]}{\sqrt{1+\text{Sin}[x]}}$$

Result (type 3, 40 leaves) :

$$\frac{2\left(-\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)\sqrt{1+\text{Sin}[x]}}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}$$

Problem 236: Result more than twice size of optimal antiderivative.

$$\int\sqrt{1-\text{Sin}[x]}\,dx$$

Optimal (type 3, 14 leaves, 1 step) :

$$\frac{2\,\text{Cos}[x]}{\sqrt{1-\text{Sin}[x]}}$$

Result (type 3, 42 leaves) :

$$\frac{2\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)\sqrt{1-\text{Sin}[x]}}{\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}$$

Problem 256: Result more than twice size of optimal antiderivative.

$$\int\frac{x}{-1+x^4}\,dx$$

Optimal (type 3, 8 leaves, 2 steps) :

$$-\frac{1}{2}\,\text{ArcTanh}\left[x^2\right]$$

Result (type 3, 23 leaves) :

$$\frac{1}{4}\,\text{Log}\left[1-x^2\right]-\frac{1}{4}\,\text{Log}\left[1+x^2\right]$$

Problem 278: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

Optimal (type 3, 94 leaves, ? steps):

$$\frac{(1+2x)\sqrt{1+2x^2+4x^3+x^4}}{2(-1+2x^2)} - \text{ArcTanh}\left[\frac{x(2+x)(7-x+27x^2+33x^3)}{(2+37x^2+31x^3)\sqrt{1+2x^2+4x^3+x^4}}\right]$$

Result (type 4, 5137 leaves):

$$\frac{(1+2x)\sqrt{1+2x^2+4x^3+x^4}}{2(-1+2x^2)} + \Big(5\,(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])^2$$

$$\Big(\left(1+\frac{1}{\sqrt{2}}\right)\text{EllipticF}\Big[\text{ArcSin}\Big[\sqrt{-\frac{(1+x)(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}\Big],$$

$$((\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\,/$$
$$((1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\Big] -$$

$$\text{EllipticPi}\Big[\frac{\left(-\frac{1}{\sqrt{2}}+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]\right)(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{\left(-1-\frac{1}{\sqrt{2}}\right)(-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])},$$

$$\text{ArcSin}\Big[\sqrt{-\frac{(1+x)(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}\Big],$$

$$((\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\,/$$
$$((1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\Big]$$

$$(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\Big)\sqrt{\frac{(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])}}$$

$$\sqrt{\frac{(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}$$

$$\sqrt{-\frac{(1+x)(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}$$

$$(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])\Big)\Big/$$

$$\Big(2\left(-1-\frac{1}{\sqrt{2}}\right)\sqrt{1+2x^2+4x^3+x^4}\left(\frac{1}{\sqrt{2}}-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]\right)(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])$$

$$\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)\Bigg)+$$

$$\Bigg(5\sqrt{2}\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)^2$$

$$\Bigg(\left(1+\frac{1}{\sqrt{2}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(1+x)\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}}\right],\right.$$

$$\left(\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,2\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)\right)\Big/$$
$$\left.\left(\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,2\right]\right)\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)\right)\right]-$$

$$\text{EllipticPi}\left[\frac{\left(-\frac{1}{\sqrt{2}}+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}{\left(-1-\frac{1}{\sqrt{2}}\right)\left(-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)},\right.$$

$$\text{ArcSin}\left[\sqrt{-\frac{(1+x)\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}}\right],$$

$$\left(\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,2\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)\right)\Big/$$
$$\left.\left(\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,2\right]\right)\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)\right)\right]$$

$$\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\Bigg)\sqrt{\frac{\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,2\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,2\right]\right)}}$$

$$\sqrt{\frac{\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}}$$

$$\sqrt{-\frac{(1+x)\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)}}$$

$$\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)\Bigg)\Big/$$

$$\left(\left(-1-\frac{1}{\sqrt{2}}\right)\sqrt{1+2x^2+4x^3+x^4}\left(\frac{1}{\sqrt{2}}-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\left(1+\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)\right.$$

$$\left(\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,3\right]\right)\Bigg)+\Bigg(5$$

$$\left(x-\text{Root}\left[1-\#1+3\#1^2+\#1^3\&,1\right]\right)^2$$

$$\left(\left(\left(1-\frac{1}{\sqrt{2}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(1+x)\,(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}\right],\right.\right.\right.$$

$$((\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\,/$$
$$\left.((1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])\,(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\right]-$$

$$\text{EllipticPi}\left[\frac{\left(\frac{1}{\sqrt{2}}+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]\right)(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{\left(-1+\frac{1}{\sqrt{2}}\right)(-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])},\right.$$

$$\text{ArcSin}\left[\sqrt{-\frac{(1+x)\,(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}\right],$$

$$((\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\,/$$
$$\left.((1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])\,(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\right]$$

$$\left.(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\right)\sqrt{\frac{(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])}}$$

$$\sqrt{\frac{(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}$$

$$\sqrt{-\frac{(1+x)\,(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}$$

$$\left.(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])\right)\Big/$$

$$\left(2\left(-1+\frac{1}{\sqrt{2}}\right)\sqrt{1+2x^2+4x^3+x^4}\left(-\frac{1}{\sqrt{2}}-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]\right)(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\right.$$
$$\left.(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])\right)-$$

$$\left(5\sqrt{2}\,(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])^2\right.$$

$$\left(\left(\left(1-\frac{1}{\sqrt{2}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(1+x)\,(\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}\right],\right.\right.\right.$$

$$((\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]-\text{Root}[1-\#1+3\#1^2+\#1^3\&,2])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3]))\,/$$

$$\left(\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)\,\left(\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)\right)\right]-$$

$$\text{EllipticPi}\left[\frac{\left(\frac{1}{\sqrt{2}}+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}{\left(-1+\frac{1}{\sqrt{2}}\right)\,\left(-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)},\right.$$

$$\text{ArcSin}\left[\sqrt{-\frac{(1+x)\,\left(\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}}\right],$$

$$\left(\left(\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)\right)\Big/$$

$$\left.\left(\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)\,\left(\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)\right)\right]$$

$$\left.\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\right)\,\sqrt{\frac{\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)}}$$

$$\sqrt{\frac{\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}}$$

$$\sqrt{-\frac{(1+x)\,\left(\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}}$$

$$\left.\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)\right)\Big/$$

$$\left(\left(-1+\frac{1}{\sqrt{2}}\right)\,\sqrt{1+2\,x^2+4\,x^3+x^4}\,\left(-\frac{1}{\sqrt{2}}-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\right.$$

$$\left.\left(\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)\right)+$$

$$\left(6\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(1+x)\,\left(-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)}}\right],\right.\right.$$

$$\left(\left(\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)\,\left(-1-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)\right)\Big/$$

$$\left.\left(\left(-1-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)\,\left(\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)\right)\right]$$

$$\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)^2\,\sqrt{\frac{\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)}{\left(x-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,1\right]\right)\,\left(1+\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,2\right]\right)}}$$

$$\left(-1-\text{Root}\left[1-\#1+3\,\#1^2+\#1^3\,\&,\,3\right]\right)$$

$$\sqrt{\frac{(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}$$

$$\left.\sqrt{\frac{(1+x)\,(-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}{(x-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])}}\right)\Big/$$

$$\left(\sqrt{1+2x^2+4x^3+x^4}\,(1+\text{Root}[1-\#1+3\#1^2+\#1^3\&,1])\,(-\text{Root}[1-\#1+3\#1^2+\#1^3\&,1]+\text{Root}[1-\#1+3\#1^2+\#1^3\&,3])\right)$$

Problem 279: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y\,(1+y)\,(2+y)\sqrt{1-y-y^2}}\,dy$$

Optimal (type 3, 142 leaves, ? steps):

$$-\frac{1}{4}\text{ArcTanh}\left[\frac{(1-3y)\sqrt{1-5y-5y^2}}{(1-5y)\sqrt{1-y-y^2}}\right]-\frac{1}{2}\text{ArcTanh}\left[\frac{(4+3y)\sqrt{1-5y-5y^2}}{(6+5y)\sqrt{1-y-y^2}}\right]+\frac{9}{4}\text{ArcTanh}\left[\frac{(11+7y)\sqrt{1-5y-5y^2}}{3\,(7+5y)\sqrt{1-y-y^2}}\right]$$

Result (type 4, 630 leaves) :

$$\frac{1}{16\sqrt{1-5y-5y^2}\sqrt{1-y-y^2}}\left(-1-\frac{2}{\sqrt{5}}\right)\left(1+\sqrt{5}+2y\right)^2\sqrt{\frac{5+3\sqrt{5}+10y}{5+5\sqrt{5}+10y}}$$

$$\left(20\left(-4\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}}\sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}}+\sqrt{5}\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}}\sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}}+5\sqrt{-\frac{-5+\sqrt{5}+2\sqrt{5}\,y}{1+\sqrt{5}+2y}}\sqrt{-\frac{-3+\sqrt{5}+2\sqrt{5}\,y}{1+\sqrt{5}+2y}}-\right.\right.$$

$$\left.2\sqrt{5}\sqrt{-\frac{-5+\sqrt{5}+2\sqrt{5}\,y}{1+\sqrt{5}+2y}}\sqrt{-\frac{-3+\sqrt{5}+2\sqrt{5}\,y}{1+\sqrt{5}+2y}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{2\sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}}\right],\frac{15}{16}\right]+$$

$$\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}}\sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}}\left(9\sqrt{5}\,\text{EllipticPi}\left[\frac{5}{8}-\frac{\sqrt{5}}{8},\text{ArcSin}\left[\frac{2\sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}}\right],\frac{15}{16}\right]+\left(-20+9\sqrt{5}\right)\right.$$

$$\left.\left.\text{EllipticPi}\left[-\frac{3}{8}\left(-5+\sqrt{5}\right),\text{ArcSin}\left[\frac{2\sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}}\right],\frac{15}{16}\right]+2\sqrt{5}\,\text{EllipticPi}\left[\frac{3}{8}\left(5+\sqrt{5}\right),\text{ArcSin}\left[\frac{2\sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}}\right],\frac{15}{16}\right]\right)\right)$$

Problem 280: Result more than twice size of optimal antiderivative.

$$\int\frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{\left(4-5x^2+x^4\right)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$$

Optimal (type 3, 21 leaves, 1 step):

$$\text{Log}\left[1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right]$$

Result (type 3, 97 leaves):

$$-\frac{1}{2}\text{ArcTanh}\left[\sqrt{-4+x^2}\right]+\frac{1}{2}\text{ArcTanh}\left[\frac{1}{2}\sqrt{-1+x^2}\right]+\frac{1}{4}\text{Log}\left[17-5x^2-4\sqrt{-4+x^2}\sqrt{-1+x^2}\right]+\frac{1}{4}\text{Log}\left[5-2x^2-2\sqrt{-4+x^2}\sqrt{-1+x^2}\right]$$

Test results for the 7 problems in "Hebisch Problems.m"

Problem 3: Unable to integrate problem.

$$\int \frac{e^{\frac{x}{2+x^2}} \left(2+2x+3x^2-x^3+2x^4\right)}{2x+x^3} \, dx$$

Optimal (type 4, 28 leaves, ? steps):

$$e^{\frac{x}{2+x^2}} \left(2+x^2\right) + \text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 43 leaves) :

$$\int \frac{e^{\frac{x}{2+x^2}} \left(2+2x+3x^2-x^3+2x^4\right)}{2x+x^3} \, dx$$

Test results for the 9 problems in "Jeffrey Problems.m"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \frac{1+\text{Cos}[x]+2\,\text{Sin}[x]}{3+\text{Cos}[x]^2+2\,\text{Sin}[x]-2\,\text{Cos}[x]\,\text{Sin}[x]} \, dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]$$

Result (type 3, 46 leaves) :

$$\frac{1}{2}\,\text{ArcTan}\left[\frac{1+\text{Cos}[x]}{-1+\text{Cos}[x]-\text{Sin}[x]}\right]-\frac{1}{2}\,\text{ArcTan}\left[\frac{1}{2}\,\text{Sec}\left[\frac{x}{2}\right]^2\left(-1+\text{Cos}[x]-\text{Sin}[x]\right)\right]$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{-5+2\,\text{Cos}[x]+7\,\text{Cos}[x]^2}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3} \, dx$$

Optimal (type 3, 25 leaves, ? steps):

$$x-2\,\text{ArcTan}\left[\frac{2\,\text{Cos}[x]\,\text{Sin}[x]}{1-\text{Cos}[x]+2\,\text{Cos}[x]^2}\right]$$

Result (type 3, 63 leaves):

$$\text{ArcTan}\left[\frac{1}{4}\text{Sec}\left[\frac{x}{2}\right]^3\left(5\,\text{Sin}\left[\frac{x}{2}\right]-3\,\text{Sin}\left[\frac{3x}{2}\right]\right)\right]-\text{ArcTan}\left[\frac{1}{4}\text{Sec}\left[\frac{x}{2}\right]^3\left(-5\,\text{Sin}\left[\frac{x}{2}\right]+3\,\text{Sin}\left[\frac{3x}{2}\right]\right)\right]$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int\frac{3}{5+4\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$x+2\,\text{ArcTan}\left[\frac{\text{Cos}[x]}{2+\text{Sin}[x]}\right]$$

Result (type 3, 79 leaves):

$$3\left(-\frac{1}{3}\text{ArcTan}\left[\frac{2\,\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+2\,\text{Sin}\left[\frac{x}{2}\right]}\right]+\frac{1}{3}\text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]+2\,\text{Sin}\left[\frac{x}{2}\right]}{2\,\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}\right]\right)$$

Test results for the 113 problems in "Moses Problems.m"

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{x^7}{1+x^{12}}\,dx$$

Optimal (type 3, 49 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2x^4}{\sqrt{3}}\right]}{4\sqrt{3}}-\frac{1}{12}\text{Log}\left[1+x^4\right]+\frac{1}{24}\text{Log}\left[1-x^4+x^8\right]$$

Result (type 3, 260 leaves):

$$\frac{1}{24}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}-2\sqrt{2}\,x}{1-\sqrt{3}}\right]-2\sqrt{3}\,\text{ArcTan}\left[\frac{1-\sqrt{3}+2\sqrt{2}\,x}{1+\sqrt{3}}\right]+\right.$$
$$2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}+2\sqrt{2}\,x}{1+\sqrt{3}}\right]-2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}+2\sqrt{2}\,x}{-1+\sqrt{3}}\right]-2\,\text{Log}\left[1-\sqrt{2}\,x+x^2\right]-2\,\text{Log}\left[1+\sqrt{2}\,x+x^2\right]+$$
$$\left.\text{Log}\left[2+\sqrt{2}\,x-\sqrt{6}\,x+2x^2\right]+\text{Log}\left[2+\sqrt{2}\left(-1+\sqrt{3}\right)x+2x^2\right]+\text{Log}\left[2-\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right]+\text{Log}\left[2+\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right]\right)$$

Problem 40: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{A^2 + B^2 - B^2 y^2}}{1 - y^2} \, dy$$

Optimal (type 3, 51 leaves, 5 steps):

$$B \operatorname{ArcTan}\left[\frac{B y}{\sqrt{A^2 + B^2 - B^2 y^2}}\right] + A \operatorname{ArcTanh}\left[\frac{A y}{\sqrt{A^2 + B^2 - B^2 y^2}}\right]$$

Result (type 3, 134 leaves):

$$-\frac{1}{2} A \operatorname{Log}[1 - y] + \frac{1}{2} A \operatorname{Log}[1 + y] + i B \operatorname{Log}\left[-2 i B y + 2 \sqrt{A^2 + B^2 - B^2 y^2}\right] + \frac{1}{2} A \operatorname{Log}\left[A^2 + B^2 - B^2 y + A \sqrt{A^2 + B^2 - B^2 y^2}\right] - \frac{1}{2} A \operatorname{Log}\left[A^2 + B^2 + B^2 y + A \sqrt{A^2 + B^2 - B^2 y^2}\right]$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[x] \sqrt{A^2 + B^2 \operatorname{Sin}[x]^2} \, dx$$

Optimal (type 3, 49 leaves, 6 steps):

$$-B \operatorname{ArcTan}\left[\frac{B \operatorname{Cos}[x]}{\sqrt{A^2 + B^2 \operatorname{Sin}[x]^2}}\right] - A \operatorname{ArcTanh}\left[\frac{A \operatorname{Cos}[x]}{\sqrt{A^2 + B^2 \operatorname{Sin}[x]^2}}\right]$$

Result (type 3, 99 leaves):

$$-\sqrt{A^2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{A^2} \operatorname{Cos}[x]}{\sqrt{2 A^2 + B^2 - B^2 \operatorname{Cos}[2 x]}}\right] + \sqrt{-B^2} \operatorname{Log}\left[\sqrt{2} \sqrt{-B^2} \operatorname{Cos}[x] + \sqrt{2 A^2 + B^2 - B^2 \operatorname{Cos}[2 x]}\right]$$

Problem 69: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int -\frac{\sqrt{A^2 + B^2 (1 - y^2)}}{1 - y^2} \, dy$$

Optimal (type 3, 53 leaves, 6 steps):

$$-B \operatorname{ArcTan}\left[\frac{B y}{\sqrt{A^2 + B^2 - B^2 y^2}}\right] - A \operatorname{ArcTanh}\left[\frac{A y}{\sqrt{A^2 + B^2 - B^2 y^2}}\right]$$

Result (type 3, 127 leaves):

$$\frac{1}{2}\left(A\,\text{Log}[1-y]-A\,\text{Log}[1+y]-2\,\text{i}\,B\,\text{Log}\left[2\left(-\text{i}\,B\,y+\sqrt{A^2+B^2-B^2\,y^2}\right)\right]-A\,\text{Log}\left[A^2+B^2-B^2\,y+A\sqrt{A^2+B^2-B^2\,y^2}\,\right]+A\,\text{Log}\left[A^2+B^2\,(1+y)+A\sqrt{A^2+B^2-B^2\,y^2}\,\right]\right)$$

Problem 70: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(-A^2-B^2\right)\text{Cos}[z]^2}{B\left(1-\frac{\left(A^2+B^2\right)\text{Sin}[z]^2}{B^2}\right)}\,dz$$

Optimal (type 3, 16 leaves, 5 steps):

$$-B\,z-A\,\text{ArcTanh}\left[\frac{A\,\text{Tan}[z]}{B}\right]$$

Result (type 3, 35 leaves):

$$-\frac{B\left(A^2+B^2\right)\left(B\,z+A\,\text{ArcTanh}\left[\frac{A\,\text{Tan}[z]}{B}\right]\right)}{A^2\,B+B^3}$$

Problem 71: Result more than twice size of optimal antiderivative.

$$\int-\frac{A^2+B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw$$

Optimal (type 3, 16 leaves, 6 steps):

$$-B\,\text{ArcTan}[w]-A\,\text{ArcTanh}\left[\frac{A\,w}{B}\right]$$

Result (type 3, 35 leaves):

$$-\frac{B\left(A^2+B^2\right)\left(B\,\text{ArcTan}[w]+A\,\text{ArcTanh}\left[\frac{A\,w}{B}\right]\right)}{A^2\,B+B^3}$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int-\frac{B\left(A^2+B^2\right)}{\left(1+w^2\right)\left(B^2-A^2\,w^2\right)}\,dw$$

Optimal (type 3, 16 leaves, 4 steps):

$$-B\,\text{ArcTan}[w]-A\,\text{ArcTanh}\left[\frac{A\,w}{B}\right]$$

Result (type 3, 35 leaves):

$$-\frac{B\left(A^2+B^2\right)\left(B\,\text{ArcTan}[w]+A\,\text{ArcTanh}\left[\frac{A\,w}{B}\right]\right)}{A^2\,B+B^3}$$

Test results for the 376 problems in "Stewart Problems.m"

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x]\,(1-\text{Sin}[x])\,dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\text{Log}[1+\text{Sin}[x]]$$

Result (type 3, 36 leaves):

$$\text{Log}[\text{Cos}[x]]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 81: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1-\text{Sin}[x]}\,dx$$

Optimal (type 3, 11 leaves, 1 step):

$$\frac{\text{Cos}[x]}{1-\text{Sin}[x]}$$

Result (type 3, 25 leaves):

$$\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}$$

Problem 97: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x]\,\text{Tan}[x]^2\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$-\frac{1}{2}\,\text{ArcTanh}[\text{Sin}[x]]+\frac{1}{2}\,\text{Sec}[x]\,\text{Tan}[x]$$

Result (type 3, 42 leaves):

$$\frac{1}{2}\left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Sec}[x]\,\text{Tan}[x]\right)$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]^4\,\text{Csc}[x]^4\,dx$$

Optimal (type 3, 17 leaves, 3 steps):

$$-\frac{1}{5}\,\text{Cot}[x]^5-\frac{\text{Cot}[x]^7}{7}$$

Result (type 3, 37 leaves):

$$-\frac{2\,\text{Cot}[x]}{35}-\frac{1}{35}\,\text{Cot}[x]\,\text{Csc}[x]^2+\frac{8}{35}\,\text{Cot}[x]\,\text{Csc}[x]^4-\frac{1}{7}\,\text{Cot}[x]\,\text{Csc}[x]^6$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[x]\,dx$$

Optimal (type 3, 5 leaves, 1 step):

$$-\text{ArcTanh}[\text{Cos}[x]]$$

Result (type 3, 17 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[x]^3\, dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$-\frac{1}{2}\text{ArcTanh}[\text{Cos}[x]]-\frac{1}{2}\text{Cot}[x]\,\text{Csc}[x]$$

Result (type 3, 47 leaves):

$$-\frac{1}{8}\text{Csc}\left[\frac{x}{2}\right]^2-\frac{1}{2}\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\frac{1}{2}\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{8}\text{Sec}\left[\frac{x}{2}\right]^2$$

Problem 104: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[x]\,\text{Cot}[x]\, dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\text{ArcTanh}[\text{Cos}[x]]+\text{Cos}[x]$$

Result (type 3, 19 leaves):

$$\text{Cos}[x]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 113: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[2\,x]\,(\text{Cos}[x]+\text{Sin}[x])\, dx$$

Optimal (type 3, 15 leaves, 6 steps):

$$-\frac{1}{2}\text{ArcTanh}[\text{Cos}[x]]+\frac{1}{2}\text{ArcTanh}[\text{Sin}[x]]$$

Result (type 3, 61 leaves):

$$-\frac{1}{2}\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]-\frac{1}{2}\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{2}\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{2}\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 121: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-a^2+x^2}}\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\text{ArcTanh}\left[\frac{x}{\sqrt{-a^2+x^2}}\right]$$

Result (type 3, 46 leaves):

$$-\frac{1}{2}\,\text{Log}\left[1-\frac{x}{\sqrt{-a^2+x^2}}\right]+\frac{1}{2}\,\text{Log}\left[1+\frac{x}{\sqrt{-a^2+x^2}}\right]$$

Problem 152: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$\text{ArcTanh}\left[\frac{x}{\sqrt{a^2+x^2}}\right]$$

Result (type 3, 42 leaves):

$$-\frac{1}{2}\,\text{Log}\left[1-\frac{x}{\sqrt{a^2+x^2}}\right]+\frac{1}{2}\,\text{Log}\left[1+\frac{x}{\sqrt{a^2+x^2}}\right]$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-x^2+x^4}\,dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$\frac{1}{x}-\text{ArcTanh}[x]$$

Result (type 3, 22 leaves):

$$\frac{1}{x}+\frac{1}{2}\,\text{Log}[1-x]-\frac{1}{2}\,\text{Log}[1+x]$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x] \left(-3+2\,\text{Sin}[x]\right)}{2-3\,\text{Sin}[x]+\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\text{Log}\left[2-3\,\text{Sin}[x]+\text{Sin}[x]^2\right]$$

Result (type 3, 26 leaves):

$$2\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}[2-\text{Sin}[x]]$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x]^2\,\text{Sin}[x]}{5+\text{Cos}[x]^2}\,dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$\sqrt{5}\,\text{ArcTan}\left[\frac{\text{Cos}[x]}{\sqrt{5}}\right]-\text{Cos}[x]$$

Result (type 3, 82 leaves):

$$\frac{1}{20}\left(-\sqrt{5}\,\text{ArcTan}\left[\frac{\text{Cos}[x]}{\sqrt{5}}\right]+21\,\sqrt{5}\,\text{ArcTan}\left[\frac{1}{\sqrt{5}}-\sqrt{\frac{6}{5}}\,\text{Tan}\left[\frac{x}{2}\right]\right]+21\,\sqrt{5}\,\text{ArcTan}\left[\frac{1}{\sqrt{5}}+\sqrt{\frac{6}{5}}\,\text{Tan}\left[\frac{x}{2}\right]\right]-20\,\text{Cos}[x]\right)$$

Problem 221: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-4\,\text{Cos}[x]+3\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$-\frac{1}{5}\,\text{ArcTanh}\left[\frac{1}{5}\left(3\,\text{Cos}[x]+4\,\text{Sin}[x]\right)\right]$$

Result (type 3, 41 leaves):

$$\frac{1}{5}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-2\,\text{Sin}\left[\frac{x}{2}\right]\right]-\frac{1}{5}\,\text{Log}\left[2\,\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 225: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x\sqrt{1+x}}\,dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-2\,\text{ArcTanh}\left[\sqrt{1+x}\right]$$

Result (type 3, 25 leaves):

$$\text{Log}\left[1-\sqrt{1+x}\right]-\text{Log}\left[1+\sqrt{1+x}\right]$$

Problem 244: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\text{Cos}[x]+\text{Sin}[x]}\,dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]-\text{Sin}[x]}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 24 leaves):

$$(-1-i)\,(-1)^{3/4}\,\text{ArcTanh}\left[\frac{-1+\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right]$$

Problem 245: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1-\text{Cos}[x]+\text{Sin}[x]}\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$-\text{Log}\left[1+\text{Cot}\left[\frac{x}{2}\right]\right]$$

Result (type 3, 24 leaves):

$$\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 246: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{4\,\text{Cos}[x] + 3\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$-\frac{1}{5}\,\text{ArcTanh}\left[\frac{1}{5}\,(3\,\text{Cos}[x] - 4\,\text{Sin}[x])\right]$$

Result (type 3, 43 leaves):

$$-\frac{1}{5}\,\text{Log}\left[2\,\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \frac{1}{5}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + 2\,\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 249: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[x]}{1 + \text{Sin}[x]}\,dx$$

Optimal (type 3, 18 leaves, 4 steps):

$$\frac{1}{2}\,\text{ArcTanh}[\text{Sin}[x]] - \frac{1}{2\,(1 + \text{Sin}[x])}$$

Result (type 3, 54 leaves):

$$\frac{1}{2}\left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] - \frac{1}{\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^2}\right)$$

Problem 255: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x]\,\text{Tan}[x]^2\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$-\frac{1}{2}\,\text{ArcTanh}[\text{Sin}[x]] + \frac{1}{2}\,\text{Sec}[x]\,\text{Tan}[x]$$

Result (type 3, 42 leaves):

$$\frac{1}{2}\left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sec}[x]\,\text{Tan}[x]\right)$$

Problem 270: Result more than twice size of optimal antiderivative.

$$\int \left(1+\sqrt{x}\right)^8 \, dx$$

Optimal (type 2, 27 leaves, 3 steps):

$$-\frac{2}{9}\left(1+\sqrt{x}\right)^9+\frac{1}{5}\left(1+\sqrt{x}\right)^{10}$$

Result (type 2, 60 leaves):

$$x+\frac{16 x^{3/2}}{3}+14 x^2+\frac{112 x^{5/2}}{5}+\frac{70 x^3}{3}+16 x^{7/2}+7 x^4+\frac{16 x^{9/2}}{9}+\frac{x^5}{5}$$

Problem 291: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-e^{-x}+e^x} \, dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\text{ArcTanh}\left[e^x\right]$$

Result (type 3, 23 leaves):

$$\frac{1}{2}\text{Log}\left[1-e^x\right]-\frac{1}{2}\text{Log}\left[1+e^x\right]$$

Problem 297: Result more than twice size of optimal antiderivative.

$$\int (1+\text{Cos}[x]) \, \text{Csc}[x] \, dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\text{Log}[1-\text{Cos}[x]]$$

Result (type 3, 20 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}[\text{Sin}[x]]$$

Problem 298: Result more than twice size of optimal antiderivative.

$$\int \frac{e^x}{-1+e^{2x}}\,dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\text{ArcTanh}\left[e^x\right]$$

Result (type 3, 23 leaves):

$$\frac{1}{2}\,\text{Log}\left[1-e^x\right]-\frac{1}{2}\,\text{Log}\left[1+e^x\right]$$

Problem 314: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[2x]^3\,\text{Csc}[2x]^3\,dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$\frac{1}{6}\,\text{Csc}[2x]^3-\frac{1}{10}\,\text{Csc}[2x]^5$$

Result (type 3, 53 leaves):

$$\frac{11\,\text{Cot}[x]}{480}+\frac{11}{960}\,\text{Cot}[x]\,\text{Csc}[x]^2-\frac{1}{320}\,\text{Cot}[x]\,\text{Csc}[x]^4+\frac{11\,\text{Tan}[x]}{480}+\frac{11}{960}\,\text{Sec}[x]^2\,\text{Tan}[x]-\frac{1}{320}\,\text{Sec}[x]^4\,\text{Tan}[x]$$

Problem 328: Result more than twice size of optimal antiderivative.

$$\int x\,\text{Sec}[x]\,\text{Tan}[x]\,dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\text{ArcTanh}[\text{Sin}[x]]+x\,\text{Sec}[x]$$

Result (type 3, 37 leaves):

$$\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+x\,\text{Sec}[x]$$

Problem 334: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-e^x + e^{3x}} \, dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$e^{-x} - \text{ArcTanh}\left[e^x\right]$$

Result (type 3, 32 leaves):

$$e^{-x} + \frac{1}{2} \text{Log}\left[1 - e^{-x}\right] - \frac{1}{2} \text{Log}\left[1 + e^{-x}\right]$$

Problem 337: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[2x]}{\sqrt{9 - \text{Cos}[x]^4}} \, dx$$

Optimal (type 3, 11 leaves, 5 steps):

$$-\text{ArcSin}\left[\frac{\text{Cos}[x]^2}{3}\right]$$

Result (type 3, 26 leaves):

$$i \, \text{Log}\left[i \, \text{Cos}[x]^2 + \sqrt{9 - \text{Cos}[x]^4}\right]$$

Problem 351: Result more than twice size of optimal antiderivative.

$$\int e^x \, \text{Sech}\left[e^x\right] \, dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\text{ArcTan}\left[\text{Sinh}\left[e^x\right]\right]$$

Result (type 3, 11 leaves):

$$2 \, \text{ArcTan}\left[\text{Tanh}\left[\frac{e^x}{2}\right]\right]$$

Problem 355: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Sec}[x]^5 \, dx$$

Optimal (type 3, 26 leaves, 3 steps):

$$\frac{3}{8} \mathrm{ArcTanh}[\mathrm{Sin}[x]] + \frac{3}{8} \mathrm{Sec}[x] \, \mathrm{Tan}[x] + \frac{1}{4} \mathrm{Sec}[x]^3 \, \mathrm{Tan}[x]$$

Result (type 3, 58 leaves):

$$\frac{1}{16} \left(-6 \, \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] - \mathrm{Sin}\left[\frac{x}{2}\right]\right] + 6 \, \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] + \mathrm{Sin}\left[\frac{x}{2}\right]\right] + \frac{1}{2} \mathrm{Sec}[x]^4 \left(11 \, \mathrm{Sin}[x] + 3 \, \mathrm{Sin}[3 \, x]\right)\right)$$

Problem 363: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{\sqrt{-2 + x^{10}}} \, dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\frac{1}{5} \mathrm{ArcTanh}\left[\frac{x^5}{\sqrt{-2 + x^{10}}}\right]$$

Result (type 3, 42 leaves):

$$-\frac{1}{10} \mathrm{Log}\left[1 - \frac{x^5}{\sqrt{-2 + x^{10}}}\right] + \frac{1}{10} \mathrm{Log}\left[1 + \frac{x^5}{\sqrt{-2 + x^{10}}}\right]$$

Problem 370: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(1 + x^3\right)^4 \, dx$$

Optimal (type 1, 11 leaves, 1 step):

$$\frac{1}{15} \left(1 + x^3\right)^5$$

Result (type 1, 36 leaves):

$$\frac{x^3}{3} + \frac{2 \, x^6}{3} + \frac{2 \, x^9}{3} + \frac{x^{12}}{3} + \frac{x^{15}}{15}$$

Test results for the 705 problems in "Timofeev Problems.m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Sec}[2\,a\,x]\,dx$$

Optimal (type 3, 13 leaves, 1 step):

$$\frac{\mathrm{ArcTanh}[\mathrm{Sin}[2\,a\,x]]}{2\,a}$$

Result (type 3, 37 leaves):

$$-\frac{\mathrm{Log}[\mathrm{Cos}[a\,x]-\mathrm{Sin}[a\,x]]}{2\,a}+\frac{\mathrm{Log}[\mathrm{Cos}[a\,x]+\mathrm{Sin}[a\,x]]}{2\,a}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{4}\,\mathrm{Csc}\left[\frac{x}{3}\right]dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$-\frac{3}{4}\,\mathrm{ArcTanh}\left[\mathrm{Cos}\left[\frac{x}{3}\right]\right]$$

Result (type 3, 23 leaves):

$$\frac{1}{4}\left(-3\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{6}\right]\right]+3\,\mathrm{Log}\left[\mathrm{Sin}\left[\frac{x}{6}\right]\right]\right)$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int -\mathrm{Sec}\left[\frac{\pi}{4}+2\,x\right]dx$$

Optimal (type 3, 15 leaves, 1 step):

$$-\frac{1}{2}\,\mathrm{ArcTanh}\left[\mathrm{Sin}\left[\frac{\pi}{4}+2\,x\right]\right]$$

Result (type 3, 55 leaves):

$$\frac{1}{2}\,\text{Log}\left[\text{Cos}\left[\frac{1}{8}\,(\pi+8\,x)\right]-\text{Sin}\left[\frac{1}{8}\,(\pi+8\,x)\right]\right]-\frac{1}{2}\,\text{Log}\left[\text{Cos}\left[\frac{1}{8}\,(\pi+8\,x)\right]+\text{Sin}\left[\frac{1}{8}\,(\pi+8\,x)\right]\right]$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{e^x}{-1+e^{2x}}\,dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\text{ArcTanh}\left[e^x\right]$$

Result (type 3, 23 leaves):

$$\frac{1}{2}\,\text{Log}\left[1-e^x\right]-\frac{1}{2}\,\text{Log}\left[1+e^x\right]$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]^3\,\text{Csc}[x]\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\text{Csc}[x]-\frac{\text{Csc}[x]^3}{3}$$

Result (type 3, 57 leaves):

$$\frac{5}{12}\,\text{Cot}\left[\frac{x}{2}\right]-\frac{1}{24}\,\text{Cot}\left[\frac{x}{2}\right]\,\text{Csc}\left[\frac{x}{2}\right]^2+\frac{5}{12}\,\text{Tan}\left[\frac{x}{2}\right]-\frac{1}{24}\,\text{Sec}\left[\frac{x}{2}\right]^2\,\text{Tan}\left[\frac{x}{2}\right]$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[x]}{1+\text{Sin}[x]}\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$x+\frac{\text{Cos}[x]}{1+\text{Sin}[x]}$$

Result (type 3, 25 leaves):

$$x-\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{-a^2+x^2}}{a}\right]}{a}$$

Result (type 3, 35 leaves):

$$-\frac{i\,\text{Log}\left[-\frac{2\,i\,a}{x}+\frac{2\sqrt{-a^2+x^2}}{x}\right]}{a}$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x-x^2}}\,dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$-\text{ArcSin}[1-2\,x]$$

Result (type 3, 38 leaves):

$$\frac{2\sqrt{-1+x}\,\sqrt{x}\,\text{Log}\left[\sqrt{-1+x}+\sqrt{x}\right]}{\sqrt{-(-1+x)\,x}}$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int \frac{1+\text{Tan}[x]^2}{1-\text{Tan}[x]^2}\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{1}{2}\,\text{ArcTanh}[2\,\text{Cos}[x]\,\text{Sin}[x]]$$

Result (type 3, 23 leaves):

$$-\frac{1}{2}\,\text{Log}[\text{Cos}[x]-\text{Sin}[x]]+\frac{1}{2}\,\text{Log}[\text{Cos}[x]+\text{Sin}[x]]$$

Problem 64: Result more than twice size of optimal antiderivative.

$$\int \left(a^2-4\,\text{Cos}[x]^2\right)^{3/4}\,\text{Sin}[2\,x]\,dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\frac{1}{7}\left(a^2-4\,\text{Cos}[x]^2\right)^{7/4}$$

Result (type 3, 127 leaves):

$$\frac{1}{7\left(-2+a^2-2\,\text{Cos}[2\,x]\right)^{1/4}}\left(6-4\,a^2+a^4-4\left(\frac{-2+a^2-2\,\text{Cos}[2\,x]}{-2+a^2}\right)^{1/4}+4\,a^2\left(\frac{-2+a^2-2\,\text{Cos}[2\,x]}{-2+a^2}\right)^{1/4}-a^4\left(\frac{-2+a^2-2\,\text{Cos}[2\,x]}{-2+a^2}\right)^{1/4}-4\left(-2+a^2\right)\,\text{Cos}[2\,x]+2\,\text{Cos}[4\,x]\right)$$

Problem 65: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[2\,x]}{\left(a^2-4\,\text{Sin}[x]^2\right)^{1/3}}\,dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$-\frac{3}{8}\left(a^2-4\,\text{Sin}[x]^2\right)^{2/3}$$

Result (type 3, 67 leaves):

$$-\frac{3\left(-2+a^2+2\,\text{Cos}[2\,x]\right)^{2/3}\left(-1+\left(\frac{-2+a^2+2\,\text{Cos}[2\,x]}{-2+a^2}\right)^{2/3}\right)}{8\left(\frac{-2+a^2+2\,\text{Cos}[2\,x]}{-2+a^2}\right)^{2/3}}$$

Problem 76: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[x]^5\,dx$$

Optimal (type 3, 26 leaves, 3 steps):

$$-\frac{3}{8}\,\text{ArcTanh}[\text{Cos}[x]]-\frac{3}{8}\,\text{Cot}[x]\,\text{Csc}[x]-\frac{1}{4}\,\text{Cot}[x]\,\text{Csc}[x]^3$$

Result (type 3, 71 leaves):

$$-\frac{3}{32}\mathrm{Csc}\left[\frac{x}{2}\right]^2-\frac{1}{64}\mathrm{Csc}\left[\frac{x}{2}\right]^4-\frac{3}{8}\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]\right]+\frac{3}{8}\mathrm{Log}\left[\mathrm{Sin}\left[\frac{x}{2}\right]\right]+\frac{3}{32}\mathrm{Sec}\left[\frac{x}{2}\right]^2+\frac{1}{64}\mathrm{Sec}\left[\frac{x}{2}\right]^4$$

Problem 99: Result more than twice size of optimal antiderivative.

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$-\frac{\mathrm{ArcTanh}\left[\frac{x}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{\mathrm{ArcTanh}\left[\frac{x}{\sqrt{3}}\right]}{\sqrt{3}}$$

Result (type 3, 69 leaves):

$$\frac{1}{12}\left(3\sqrt{2}\,\mathrm{Log}\left[\sqrt{2}-x\right]+2\sqrt{3}\,\mathrm{Log}\left[\sqrt{3}-x\right]-3\sqrt{2}\,\mathrm{Log}\left[\sqrt{2}+x\right]-2\sqrt{3}\,\mathrm{Log}\left[\sqrt{3}+x\right]\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+x^2+x^4}\,dx$$

Optimal (type 3, 67 leaves, 9 steps):

$$-\frac{\mathrm{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{\mathrm{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{4}\mathrm{Log}\left[1-x+x^2\right]+\frac{1}{4}\mathrm{Log}\left[1+x+x^2\right]$$

Result (type 3, 73 leaves):

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\,\mathrm{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]-\sqrt{1+i\sqrt{3}}\,\mathrm{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]\right)}{\sqrt{6}}$$

Problem 193: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b1+c1\,x)\left(a+2\,b\,x+c\,x^2\right)^n\,dx$$

Optimal (type 5, 159 leaves, 2 steps):

$$\frac{c1\left(a+2bx+cx^2\right)^{1+n}}{2c(1+n)}-\frac{2^n(b1\,c-b\,c1)\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1-n}\left(a+2bx+cx^2\right)^{1+n}\text{Hypergeometric2F1}\left[-n,1+n,2+n,\frac{b+\sqrt{b^2-ac}+cx}{2\sqrt{b^2-ac}}\right]}{c\sqrt{b^2-ac}\,(1+n)}$$

Result (type 6, 471 leaves):

$$\frac{1}{2}\left(b-\sqrt{b^2-ac}+cx\right)\left(a+x(2b+cx)\right)^n$$
$$\left(\left(3\left(b+\sqrt{b^2-ac}\right)c1\,x^2\left(a+\left(b-\sqrt{b^2-ac}\right)x\right)^2\text{AppellF1}\left[2,-n,-n,3,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right]\right)\Big/\right.$$
$$\left(\left(-b+\sqrt{b^2-ac}\right)\left(b+\sqrt{b^2-ac}+cx\right)\left(a+x(2b+cx)\right)\left(-3a\,\text{AppellF1}\left[2,-n,-n,3,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right]+\right.\right.$$
$$nx\left(\left(-b+\sqrt{b^2-ac}\right)\text{AppellF1}\left[3,1-n,-n,4,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right]-\left(b+\sqrt{b^2-ac}\right)\text{AppellF1}\left[3,-n,1-n,4,\right.\right.$$
$$\left.\left.\left.\left.-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right]\right)\right)\right)+\frac{2^{1+n}b1\left(\frac{b+\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-n}\text{Hypergeometric2F1}\left[-n,1+n,2+n,\frac{-b+\sqrt{b^2-ac}-cx}{2\sqrt{b^2-ac}}\right]}{c(1+n)}\right)$$

Problem 198: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int(b1+c1\,x)\left(a+2bx+cx^2\right)^{-n}dx$$

Optimal (type 5, 169 leaves, 2 steps):

$$\frac{c1\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(b1\,c-b\,c1)\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1+n}\left(a+2bx+cx^2\right)^{1-n}\text{Hypergeometric2F1}\left[1-n,n,2-n,\frac{b+\sqrt{b^2-ac}+cx}{2\sqrt{b^2-ac}}\right]}{c\sqrt{b^2-ac}\,(1-n)}$$

Result (type 6, 374 leaves):

$$\frac{1}{2}\left(a+x\left(2b+cx\right)\right)^{-n}\left(-\left(\left(3\,a\,c1\,x^2\,\text{AppellF1}\left[2,n,n,3,-\frac{c\,x}{b+\sqrt{b^2-a\,c}},\frac{c\,x}{-b+\sqrt{b^2-a\,c}}\right]\right)\Big/\left(-3\,a\,\text{AppellF1}\left[2,n,n,3,-\frac{c\,x}{b+\sqrt{b^2-a\,c}},\frac{c\,x}{-b+\sqrt{b^2-a\,c}}\right]+n\,x\left(\left(b+\sqrt{b^2-a\,c}\right)\text{AppellF1}\left[3,n,1+n,4,-\frac{c\,x}{b+\sqrt{b^2-a\,c}},\frac{c\,x}{-b+\sqrt{b^2-a\,c}}\right]+\left(b-\sqrt{b^2-a\,c}\right)\text{AppellF1}\left[3,1+n,n,4,-\frac{c\,x}{b+\sqrt{b^2-a\,c}},\frac{c\,x}{-b+\sqrt{b^2-a\,c}}\right]\right)\right)\right)-\frac{2^{1-n}\,b1\left(b-\sqrt{b^2-a\,c}+c\,x\right)\left(\frac{b+\sqrt{b^2-a\,c}+c\,x}{\sqrt{b^2-a\,c}}\right)^n\text{Hypergeometric2F1}\left[1-n,n,2-n,\frac{-b+\sqrt{b^2-a\,c}-c\,x}{2\sqrt{b^2-a\,c}}\right]}{c\left(-1+n\right)}\right)$$

Problem 217: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(-1+x\right)^{2/3}x^5}\,dx$$

Optimal (type 3, 104 leaves, 8 steps):

$$\frac{\left(-1+x\right)^{1/3}}{4\,x^4}+\frac{11\left(-1+x\right)^{1/3}}{36\,x^3}+\frac{11\left(-1+x\right)^{1/3}}{27\,x^2}+\frac{55\left(-1+x\right)^{1/3}}{81\,x}-\frac{110\,\text{ArcTan}\left[\frac{1-2\left(-1+x\right)^{1/3}}{\sqrt{3}}\right]}{81\sqrt{3}}+\frac{55}{81}\text{Log}\left[1+\left(-1+x\right)^{1/3}\right]-\frac{55\,\text{Log}[x]}{243}$$

Result (type 5, 63 leaves):

$$\frac{-81-18\,x-33\,x^2-88\,x^3+220\,x^4-220\left(\frac{-1+x}{x}\right)^{2/3}x^4\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{x}\right]}{324\left(-1+x\right)^{2/3}x^4}$$

Problem 221: Result unnecessarily involves higher level functions.

$$\int\frac{x^2\sqrt{1+x}\,\left(1-x^2\right)^{1/4}}{\sqrt{1-x}\,\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$$

Optimal (type 3, 304 leaves, 33 steps):

$$\frac{5}{16}(1-x)^{3/4}(1+x)^{1/4}-\frac{1}{16}(1-x)^{1/4}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{7\left(1-x^2\right)^{5/4}}{24\sqrt{1-x}}+\frac{x\left(1-x^2\right)^{5/4}}{6\sqrt{1-x}}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}-$$
$$\frac{3\,\text{ArcTan}\left[1-\frac{\sqrt{2}(1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8\sqrt{2}}+\frac{3\,\text{ArcTan}\left[1+\frac{\sqrt{2}(1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{1-x}}{\sqrt{1+x}}-\frac{\sqrt{2}(1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8\sqrt{2}}-\frac{\text{Log}\left[1+\frac{\sqrt{1-x}}{\sqrt{1+x}}+\frac{\sqrt{2}(1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8\sqrt{2}}$$

Result (type 5, 165 leaves):

$$-\frac{1}{48}\sqrt{1+x}\left(1-x^2\right)^{1/4}\left(-7+2x+8x^2-\frac{\sqrt{1-x^2}\left(29+22x+8x^2\right)}{1+x}\right)+$$
$$\frac{\left(-2(-1+x)-(-1+x)^2\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1-x}{2}\right]}{8\times 2^{1/4}(1+x)^{1/4}}+\frac{5\left(-2(-1+x)-(-1+x)^2\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1-x}{2}\right]}{24\times 2^{3/4}(1+x)^{3/4}}$$

Problem 222: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{1-x}\;x\;(1+x)^{2/3}}{-(1-x)^{5/6}(1+x)^{1/3}+(1-x)^{2/3}\sqrt{1+x}}\,dx$$

Optimal (type 3, 292 leaves, ? steps):

$$-\frac{1}{12}(1-3x)(1-x)^{2/3}(1+x)^{1/3}+\frac{1}{4}\sqrt{1-x}\;x\sqrt{1+x}-\frac{1}{4}(1-x)(3+x)+$$
$$\frac{1}{12}(1-x)^{1/3}(1+x)^{2/3}(1+3x)+\frac{1}{12}(1-x)^{1/6}(1+x)^{5/6}(2+3x)-\frac{1}{12}(1-x)^{5/6}(1+x)^{1/6}(10+3x)+$$
$$\frac{1}{6}\text{ArcTan}\left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}}\right]-\frac{4\,\text{ArcTan}\left[\frac{(1-x)^{1/3}-2(1+x)^{1/3}}{\sqrt{3}(1-x)^{1/3}}\right]}{3\sqrt{3}}-\frac{5}{6}\text{ArcTan}\left[\frac{(1-x)^{1/3}-(1+x)^{1/3}}{(1-x)^{1/6}(1+x)^{1/6}}\right]+\frac{\text{ArcTanh}\left[\frac{\sqrt{3}(1-x)^{1/6}(1+x)^{1/6}}{(1-x)^{1/3}+(1+x)^{1/3}}\right]}{6\sqrt{3}}$$

Result (type 5, 391 leaves):

$$-\frac{2^{2/3}\left(-2(-1+x)-(-1+x)^2\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1-x}{2}\right]}{3(1+x)^{1/3}}-$$

$$\frac{1}{12}(1+x)^{1/3}\left((1-3x)(1-x)^{2/3}-\frac{3(1-x)^{1/3}x(2+x)}{(1-x^2)^{1/3}}-3(1-x)^{1/3}x(1-x^2)^{1/6}-(1+3x)(1-x^2)^{1/3}-\frac{(2+3x)\sqrt{1-x^2}}{(1-x)^{1/3}}+\frac{(10+3x)(1-x^2)^{5/6}}{1+x}-\right.$$

$$\left.4\times 2^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1+x}{2}\right]\right)-\frac{7\left(-2(-1+x)-(-1+x)^2\right)^{5/6}\text{Hypergeometric2F1}\left[\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1-x}{2}\right]}{30\times 2^{5/6}(1+x)^{5/6}}+$$

$$\frac{(1+x)^{1/3}\sqrt{2(1+x)-(1+x)^2}\,\text{Hypergeometric2F1}\left[\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1+x}{2}\right]}{6\times 2^{5/6}\sqrt{1-x}}+\frac{(1-x)^{1/3}\sqrt{-1+x}\,(1+x)^{5/6}\,\text{Log}\left[\sqrt{-1+x}+\sqrt{1+x}\right]}{2\left(2(1+x)-(1+x)^2\right)^{5/6}}$$

Problem 226: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left((-1+x)^2(1+x)\right)^{1/3}}\,dx$$

Optimal (type 3, 67 leaves, ? steps):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\text{Log}[1+x]-\frac{3}{2}\text{Log}\left[1-\frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]$$

Result (type 5, 49 leaves) :

$$\frac{3(-1+x)(1+x)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1-x}{2}\right]}{2^{1/3}\left((-1+x)^2(1+x)\right)^{1/3}}$$

Problem 228: Result unnecessarily involves higher level functions.

$$\int \frac{\left((-1+x)^2(1+x)\right)^{1/3}}{x^2}\,dx$$

Optimal (type 3, 150 leaves, ? steps):

$$-\frac{\left((-1+x)^2 (1+x)\right)^{1/3}}{x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2 (-1+x)}{\left((-1+x)^2 (1+x)\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2 (-1+x)}{\left((-1+x)^2 (1+x)\right)^{1/3}}}{\sqrt{3}}\right]+$$
$$\frac{\text{Log}[x]}{6}-\frac{2}{3}\text{Log}[1+x]-\frac{3}{2}\text{Log}\left[1-\frac{-1+x}{\left((-1+x)^2 (1+x)\right)^{1/3}}\right]-\frac{1}{2}\text{Log}\left[1+\frac{-1+x}{\left((-1+x)^2 (1+x)\right)^{1/3}}\right]$$

Result (type 6, 145 leaves) :

$$\frac{1}{2}\left((-1+x)^2 (1+x)\right)^{1/3}\left(-\frac{2}{x}-\left(4\,x\,\text{AppellF1}\left[1,\frac{1}{3},\frac{2}{3},2,\frac{1}{x},-\frac{1}{x}\right]\right)\Big/\right.$$
$$\left((-1+x)(1+x)\left(6\,x\,\text{AppellF1}\left[1,\frac{1}{3},\frac{2}{3},2,\frac{1}{x},-\frac{1}{x}\right]-2\,\text{AppellF1}\left[2,\frac{1}{3},\frac{5}{3},3,\frac{1}{x},-\frac{1}{x}\right]+\text{AppellF1}\left[2,\frac{4}{3},\frac{2}{3},3,\frac{1}{x},-\frac{1}{x}\right]\right)\right)-$$
$$\left.\frac{3\times 2^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1+x}{2}\right]}{(1-x)^{2/3}}\right)$$

Problem 232: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(9+3x-5x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 75 leaves, ? steps):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2 (-3+x)}{\left(9+3x-5x^2+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\text{Log}[1+x]-\frac{3}{2}\text{Log}\left[1-\frac{-3+x}{\left(9+3x-5x^2+x^3\right)^{1/3}}\right]$$

Result (type 5, 49 leaves) :

$$\frac{3(-3+x)(1+x)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{3-x}{4}\right]}{2^{2/3}\left((-3+x)^2 (1+x)\right)^{1/3}}$$

Problem 245: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x}{\left(4+x+x^2\right)\sqrt{5+4x+4x^2}}\,dx$$

Optimal (type 3, 63 leaves, 5 steps) :

$$\frac{\text{ArcTan}\left[\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right]}{\sqrt{11}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{15}}\,(1+2x)}{\sqrt{5+4x+4x^2}}\right]}{\sqrt{165}}$$

Result (type 3, 426 leaves):

$$\frac{\left(\text{i}+\sqrt{15}\right)\text{ArcTan}\left[\frac{-4\sqrt{15}-\sqrt{15}\,x-\sqrt{15}\,x^2-4\sqrt{11}\,\sqrt{5+4x+4x^2}}{16+15x+15x^2}\right]}{2\sqrt{165}}+\frac{\left(-\text{i}+\sqrt{15}\right)\text{ArcTan}\left[\frac{4\sqrt{15}+\sqrt{15}\,x+\sqrt{15}\,x^2-4\sqrt{11}\,\sqrt{5+4x+4x^2}}{16+15x+15x^2}\right]}{2\sqrt{165}}+$$
$$\frac{\text{i}\left(-\text{i}+\sqrt{15}\right)\text{Log}\left[\left(-\text{i}+\sqrt{15}-2\,\text{i}\,x\right)^2\left(\text{i}+\sqrt{15}+2\,\text{i}\,x\right)^2\right]}{4\sqrt{165}}+\frac{\text{i}\left(\text{i}+\sqrt{15}\right)\text{Log}\left[\left(-\text{i}+\sqrt{15}-2\,\text{i}\,x\right)^2\left(\text{i}+\sqrt{15}+2\,\text{i}\,x\right)^2\right]}{4\sqrt{165}}-$$
$$\frac{\text{i}\left(\text{i}+\sqrt{15}\right)\text{Log}\left[\left(4+x+x^2\right)\left(43+52x+52x^2-\sqrt{165}\,\sqrt{5+4x+4x^2}-2\sqrt{165}\,x\,\sqrt{5+4x+4x^2}\right)\right]}{4\sqrt{165}}-$$
$$\frac{\text{i}\left(-\text{i}+\sqrt{15}\right)\text{Log}\left[\left(4+x+x^2\right)\left(43+52x+52x^2+\sqrt{165}\,\sqrt{5+4x+4x^2}+2\sqrt{165}\,x\,\sqrt{5+4x+4x^2}\right)\right]}{4\sqrt{165}}$$

Problem 246: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{3+x}{\left(1+x^2\right)\sqrt{1+x+x^2}}\,dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$-2\sqrt{2}\,\text{ArcTan}\left[\frac{1-x}{\sqrt{2}\,\sqrt{1+x+x^2}}\right]+\sqrt{2}\,\text{ArcTanh}\left[\frac{1+x}{\sqrt{2}\,\sqrt{1+x+x^2}}\right]$$

Result (type 3, 352 leaves):

$$\left(\frac{1}{4}+\frac{\text{i}}{4}\right)(-1)^{3/4}\Bigg((4+2\,\text{i})\,\text{ArcTan}\Big[\Big((-7+12\,\text{i})+(12+25\,\text{i})\,x^3+40\,(-1)^{1/4}\sqrt{1+x+x^2}+x^2\left((5+28\,\text{i})+20\,(-1)^{3/4}\sqrt{1+x+x^2}\right)+$$
$$x\left((-4+37\,\text{i})-(10-30\,\text{i})\sqrt{2}\,\sqrt{1+x+x^2}\right)\Big)\Big/\left((1-36\,\text{i})+(32-11\,\text{i})\,x+(5+16\,\text{i})\,x^2+(4+25\,\text{i})\,x^3\right)\Big]+$$
$$(4-2\,\text{i})\,\text{ArcTan}\Big[\Big((-7-12\,\text{i})+(12-25\,\text{i})\,x^3+20\,(-1)^{1/4}\sqrt{1+x+x^2}+x^2\left((5-28\,\text{i})-40\,(-1)^{3/4}\sqrt{1+x+x^2}\right)+$$
$$x\left((-4-37\,\text{i})+(30+10\,\text{i})\sqrt{2}\,\sqrt{1+x+x^2}\right)\Big)\Big/\left((-49+36\,\text{i})-(48-61\,\text{i})\,x-(45-64\,\text{i})\,x^2+(4+25\,\text{i})\,x^3\right)\Big]+$$
$$2\,\text{Log}\left[1+x^2\right]-(1+2\,\text{i})\,\text{Log}\left[(5+4\,\text{i})+(8+4\,\text{i})\,x+(5+4\,\text{i})\,x^2+8\,(-1)^{1/4}\sqrt{1+x+x^2}+4\,(-1)^{1/4}x\sqrt{1+x+x^2}\right]-$$
$$(1-2\,\text{i})\,\text{Log}\left[(5+4\,\text{i})+(8+4\,\text{i})\,x+(5+4\,\text{i})\,x^2+4\,(-1)^{1/4}\sqrt{1+x+x^2}+8\,(-1)^{1/4}x\sqrt{1+x+x^2}\right]\Bigg)$$

Problem 247: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,\left(4+4x+3x^2\right)}\,dx$$

Optimal (type 3, 70 leaves, 5 steps):

$$-\frac{5\,\text{ArcTan}\left[\frac{\sqrt{\frac{7}{2}}\,(2-x)}{2\sqrt{-1+6x+x^2}}\right]}{6\sqrt{14}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{7}\,(1+x)}{\sqrt{-1+6x+x^2}}\right]}{3\sqrt{7}}$$

Result (type 3, 991 leaves):

$$\frac{1}{8\sqrt{14}}\left(\frac{1}{\sqrt{7+4\,i\,\sqrt{2}}}2\left(-i+4\sqrt{2}\right)\text{ArcTan}\Big[\Big(7840-5816\,i\,\sqrt{2}+18\left(112+37\,i\,\sqrt{2}\right)x^4+3564\,i\,\sqrt{7\left(7+4\,i\,\sqrt{2}\right)}\,\sqrt{-1+6x+x^2}+\right.$$

$$i\,x^2\left(56224\,i+29126\sqrt{2}-99\sqrt{7\left(7+4\,i\,\sqrt{2}\right)}\,\sqrt{-1+6x+x^2}\right)-72\,i\,x\left(-546\,i+265\sqrt{2}-11\sqrt{7\left(7+4\,i\,\sqrt{2}\right)}\,\sqrt{-1+6x+x^2}\right)+$$

$$3x^3\left(1456+7564\,i\,\sqrt{2}-693\,i\,\sqrt{7\left(7+4\,i\,\sqrt{2}\right)}\,\sqrt{-1+6x+x^2}\right)\Big)\Big/\Big(9836\,i-5600\sqrt{2}+36\left(-1083\,i+560\sqrt{2}\right)x+$$

$$\left(-41651\,i+78176\sqrt{2}\right)x^2+\left(-91506\,i+61824\sqrt{2}\right)x^3+9\left(-1487\,i+896\sqrt{2}\right)x^4\Big)\Big]-\frac{1}{\sqrt{-7+4\,i\,\sqrt{2}}}$$

$$2\left(i+4\sqrt{2}\right)\text{ArcTanh}\Big[\Big(4\left(6344\,i-700\sqrt{2}+18\left(477\,i+140\sqrt{2}\right)x+\left(9847\,i+9772\sqrt{2}\right)x^2+12\left(947\,i+644\sqrt{2}\right)x^3+9\left(421\,i+112\sqrt{2}\right)x^4\right)\Big)\Big/$$

$$\Big(-9\left(112\,i+37\sqrt{2}\right)x^4+36x\left(546\,i+265\sqrt{2}+44\sqrt{-98+56\,i\,\sqrt{2}}\,\sqrt{-1+6x+x^2}\right)+$$

$$3x^3\left(-728\,i-3782\sqrt{2}+99\sqrt{-98+56\,i\,\sqrt{2}}\,\sqrt{-1+6x+x^2}\right)+4\left(-980\,i+727\sqrt{2}+297\sqrt{-98+56\,i\,\sqrt{2}}\,\sqrt{-1+6x+x^2}\right)+$$

$$x^2\left(28112\,i-14563\sqrt{2}+1287\sqrt{-98+56\,i\,\sqrt{2}}\,\sqrt{-1+6x+x^2}\right)\Big)\Big]+$$

$$\frac{\left(1+4\,i\,\sqrt{2}\right)\text{Log}\left[9\left(4+4x+3x^2\right)^2\right]}{\sqrt{7+4\,i\,\sqrt{2}}}+\frac{\left(i+4\sqrt{2}\right)\text{Log}\left[9\left(4+4x+3x^2\right)^2\right]}{\sqrt{-7+4\,i\,\sqrt{2}}}-\frac{1}{\sqrt{-7+4\,i\,\sqrt{2}}}$$

$$\left(i+4\sqrt{2}\right)\text{Log}\Big[\left(4+4x+3x^2\right)$$

$$\left(-101\,i-14\sqrt{2}+2\left(-2\,i+7\sqrt{2}\right)x^2+9\,i\,\sqrt{7\left(-7+4\,i\,\sqrt{2}\right)}\,\sqrt{-1+6x+x^2}+x\left(186\,i+84\sqrt{2}-7\,i\,\sqrt{7\left(-7+4\,i\,\sqrt{2}\right)}\,\sqrt{-1+6x+x^2}\right)\right)\Big]-$$

$$\frac{1}{\sqrt{7+4\,i\,\sqrt{2}}}i\left(-i+4\sqrt{2}\right)\text{Log}\Big[\left(4+4x+3x^2\right)\Big(\left(-53\,i+14\sqrt{2}\right)x^2+2x\left(-54\,i+42\sqrt{2}-i\sqrt{98+56\,i\,\sqrt{2}}\,\sqrt{-1+6x+x^2}\right)-$$

$$\left.2\,i\left(26-7\,i\,\sqrt{2}+3\sqrt{98+56\,i\,\sqrt{2}}\,\sqrt{-1+6x+x^2}\right)\Big)\Big]\right)$$

Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{B+A\,x}{\left(17-18x+5x^2\right)\sqrt{13-22x+10x^2}}\,dx$$

Optimal (type 3, 80 leaves, 5 steps):

$$-\frac{(2A+B)\,\text{ArcTan}\left[\frac{\sqrt{35}\,(2-x)}{\sqrt{13-22x+10x^2}}\right]}{\sqrt{35}}-\frac{(A+B)\,\text{ArcTanh}\left[\frac{\sqrt{35}\,(1-x)}{2\sqrt{13-22x+10x^2}}\right]}{2\sqrt{35}}$$

Result (type 3, 1149 leaves) :

$$\frac{1}{8\sqrt{35}}\Bigg(\Big(\big((8-2i)A+(4-2i)B\big)$$
$$\text{ArcTan}\Big[\Big(4A^2\big((-2494-6746i)+(3811+15444i)x-(1900+11640i)x^2+(300+2800i)x^3\big)+(2+4i)B^2\big((-1843+92i)+(3955+186i)x-$$
$$(2827+336i)x^2+(645+110i)x^3\big)+(4+8i)AB\big((-3439-76i)+(7427+942i)x-(5354+1092i)x^2+(1240+320i)x^3\big)\Big)\Big/$$
$$\Big((1+2i)B^2\Big((-608-1208i)+(395+610i)x^3+(66-77i)\sqrt{35}\sqrt{13-22x+10x^2}+$$
$$x\left((1540+3036i)-(104-103i)\sqrt{35}\sqrt{13-22x+10x^2}\right)+(4-3i)x^2\left((80-549i)+10\sqrt{35}\sqrt{13-22x+10x^2}\right)\Big)+$$
$$A^2\Big((10987-3210i)-(4800-2800i)x^3+(748+187i)\sqrt{35}\sqrt{13-22x+10x^2}+$$
$$10x^2\left((1969-892i)+(34+17i)\sqrt{35}\sqrt{13-22x+10x^2}\right)-x\left((25633-9460i)+(1054+357i)\sqrt{35}\sqrt{13-22x+10x^2}\right)\Big)+$$
$$AB\Big((9519-6362i)-(4225-4200i)x^3+(792+198i)\sqrt{35}\sqrt{13-22x+10x^2}+$$
$$(10+5i)x^2\left((828-1871i)+36\sqrt{35}\sqrt{13-22x+10x^2}\right)-x\left((22801-16808i)+(1116+378i)\sqrt{35}\sqrt{13-22x+10x^2}\right)\Big)\Big)\Big]+$$
$$\Big(\big((-2+8i)A-(2-4i)B\big)\text{ArcTanh}\Big[\Big(A^2\big((3594-15991i)-(8096-43289i)x+(5990-39425i)x^2-(1400-12175i)x^3\big)+$$
$$(2+i)B^2\big((-367-3288i)+(1085+8506i)x-(1073+7336i)x^2+(355+2110i)x^3\big)+$$
$$(4+2i)AB\big((-1147-4952i)+(3185+12882i)x-(2993+11256i)x^2+(955+3310i)x^3\big)\Big)\Big/$$
$$\Big(2\Big((2+i)B^2\Big((-1208-608i)+(610+395i)x^3+(77-66i)\sqrt{35}\sqrt{13-22x+10x^2}+$$
$$x\left((3036+1540i)-(103-104i)\sqrt{35}\sqrt{13-22x+10x^2}\right)+(3-4i)x^2\left((-80-549i)+10\sqrt{35}\sqrt{13-22x+10x^2}\right)\Big)+$$
$$AB\Big((-9519-6362i)+(4225+4200i)x^3+(792-198i)\sqrt{35}\sqrt{13-22x+10x^2}+$$
$$x\left((22801+16808i)-(1116-378i)\sqrt{35}\sqrt{13-22x+10x^2}\right)+(10-5i)x^2\left((-828-1871i)+36\sqrt{35}\sqrt{13-22x+10x^2}\right)\Big)+$$
$$A^2\Big((4800+2800i)x^3+x\left((25633+9460i)-(1054-357i)\sqrt{35}\sqrt{13-22x+10x^2}\right)-$$
$$10ix^2\left((892-1969i)+(17+34i)\sqrt{35}\sqrt{13-22x+10x^2}\right)+(1-4i)\left((109-2774i)+(88+165i)\sqrt{35}\sqrt{13-22x+10x^2}\right)\Big)\Big)\Big)\Big]-$$
$$2A\,\text{Log}\left[i\left(17-18x+5x^2\right)\right]-2B\,\text{Log}\left[i\left(17-18x+5x^2\right)\right]+(1-4i)$$
$$A$$
$$\text{Log}\Big[$$

$$(1+2\,i)\left((-127-1566\,i)+(118+2844\,i)\,x-(25+1350\,i)\,x^2+68\,i\sqrt{35}\sqrt{13-22x+10x^2}-70\,i\sqrt{35}\,x\sqrt{13-22x+10x^2}\right)]+$$

$$(1-2\,i)\,B\,\text{Log}[(1+2\,i)\left((-127-1566\,i)+(118+2844\,i)\,x-(25+1350\,i)\,x^2+68\,i\sqrt{35}\sqrt{13-22x+10x^2}-70\,i\sqrt{35}\,x\sqrt{13-22x+10x^2}\right)]+$$

$$(1+4\,i)\,A\,\text{Log}[$$

$$(2+i)\left((1566+127\,i)-(2844+118\,i)\,x+(1350+25\,i)\,x^2-68\sqrt{35}\sqrt{13-22x+10x^2}+70\sqrt{35}\,x\sqrt{13-22x+10x^2}\right)]+$$

$$(1+2\,i)\,B\,\text{Log}[(2+i)\left((1566+127\,i)-(2844+118\,i)\,x+(1350+25\,i)\,x^2-68\sqrt{35}\sqrt{13-22x+10x^2}+70\sqrt{35}\,x\sqrt{13-22x+10x^2}\right)])$$

Problem 249: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{35}\,(1-x)}{2\sqrt{13-22x+10x^2}}\right]}{2\sqrt{35}}$$

Result (type 3, 410 leaves):

$$\frac{1}{8\sqrt{35}}\left(-2\,i\,\text{ArcTan}\left[\left(4\left((-2+2\,i)+5x\right)(13-22x+10x^2)\right)\Big/\left((-819-182\,i)+350x^3+(44+11\,i)\sqrt{35}\sqrt{13-22x+10x^2}+\right.\right.\right.$$

$$\left.\left. x\left((1841+308\,i)-(62+21\,i)\sqrt{35}\sqrt{13-22x+10x^2}\right)+10x^2\left((-140-14\,i)+(2+i)\sqrt{35}\sqrt{13-22x+10x^2}\right)\right)\right]-$$

$$2\,i\,\text{ArcTan}\left[\left((7+14\,i)\left((-169-116\,i)+(419+218\,i)\,x-(335+140\,i)\,x^2+(85+30\,i)\,x^3\right)\right)\Big/\right.$$

$$\left((-1638+364\,i)+700x^3-(88-22\,i)\sqrt{35}\sqrt{13-22x+10x^2}+20\,i\,x^2\left((14+140\,i)+(1+2\,i)\sqrt{35}\sqrt{13-22x+10x^2}\right)+\right.$$

$$\left.\left.(4-2\,i)\,x\left((798+245\,i)+(29+4\,i)\sqrt{35}\sqrt{13-22x+10x^2}\right)\right)\right]+2\,\text{Log}\left[i\,(17-18x+5x^2)\right]-$$

$$\text{Log}\left[(1+2\,i)\left((1566-127\,i)-(2844-118\,i)\,x+(1350-25\,i)\,x^2-68\sqrt{35}\sqrt{13-22x+10x^2}+70\sqrt{35}\,x\sqrt{13-22x+10x^2}\right)\right]-$$

$$\left.\text{Log}\left[(2+i)\left((1566+127\,i)-(2844+118\,i)\,x+(1350+25\,i)\,x^2-68\sqrt{35}\sqrt{13-22x+10x^2}+70\sqrt{35}\,x\sqrt{13-22x+10x^2}\right)\right]\right)$$

Problem 260: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^2 \left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2} \left(1-x^3+\left(1+x^2\right)^{3/2}\right)} \, dx$$

Optimal (type 3, 136 leaves, 32 steps):

$$\frac{8 x}{9}-\frac{x^2}{6}+\frac{8 \sqrt{1+x^2}}{9}-\frac{1}{6} x \sqrt{1+x^2}-\frac{41 \text{ArcSinh}[x]}{54}+\frac{4}{27} \sqrt{2} \text{ArcTan}\left[\frac{1+3 x}{2 \sqrt{2}}\right]+\frac{4}{27} \sqrt{2} \text{ArcTan}\left[\frac{1+x}{\sqrt{2} \sqrt{1+x^2}}\right]+\frac{7}{27} \text{ArcTanh}\left[\frac{1-x}{2 \sqrt{1+x^2}}\right]-\frac{7}{54} \text{Log}\left[3+2 x+3 x^2\right]$$

Result (type 3, 947 leaves):

$$\frac{1}{108} \left(96 x-18 x^2-6 (-16+3 x) \sqrt{1+x^2}-82 \text{ArcSinh}[x]+16 \sqrt{2} \text{ArcTan}\left[\frac{1+3 x}{2 \sqrt{2}}\right]-\frac{1}{\sqrt{1+2 i \sqrt{2}}} 2 i \left(-i+11 \sqrt{2}\right) \text{ArcTan}\left[\left(2 \left(169 \left(7-4 i \sqrt{2}\right)-1716 i \left(-i+2 \sqrt{2}\right) x+\left(-4622-5032 i \sqrt{2}\right) x^2-1716 i \left(-i+2 \sqrt{2}\right) x^3+\left(-1449-4356 i \sqrt{2}\right) x^4\right)\right) \Big/ \left(-559 \left(-8 i+7 \sqrt{2}\right)+9 \left(-88 i+383 \sqrt{2}\right) x^4+12 x \left(230 \left(4 i+\sqrt{2}\right)+729 \sqrt{1+2 i \sqrt{2}} \sqrt{1+x^2}\right)+12 x^3 \left(230 \left(4 i+\sqrt{2}\right)+729 \sqrt{1+2 i \sqrt{2}} \sqrt{1+x^2}\right)+x^2 \left(3680 i-862 \sqrt{2}+5832 \sqrt{1+2 i \sqrt{2}} \sqrt{1+x^2}\right)\right)\right]+\frac{1}{\sqrt{-1+2 i \sqrt{2}}} 2 \left(i+11 \sqrt{2}\right) \text{ArcTan}\left[\left(559 \left(8-7 i \sqrt{2}\right)+9 i \left(88 i+383 \sqrt{2}\right) x^4+6561 i \sqrt{-2+4 i \sqrt{2}} \sqrt{1+x^2}+3 x^3 \left(920 \left(4+i \sqrt{2}\right)-729 i \sqrt{-2+4 i \sqrt{2}} \sqrt{1+x^2}\right)+3 x \left(920 \left(4+i \sqrt{2}\right)+729 i \sqrt{-2+4 i \sqrt{2}} \sqrt{1+x^2}\right)+x^2 \left(3680-862 i \sqrt{2}+5103 i \sqrt{-2+4 i \sqrt{2}} \sqrt{1+x^2}\right)\right) \Big/ \left(17317 i+1352 \sqrt{2}+3432 \left(i+2 \sqrt{2}\right) x+2 \left(19931 i+5032 \sqrt{2}\right) x^2+3432 \left(i+2 \sqrt{2}\right) x^3+9 \left(2509 i+968 \sqrt{2}\right) x^4\right)\right]-14 \text{Log}\left[3+2 x+3 x^2\right]-\frac{\left(-i+11 \sqrt{2}\right) \text{Log}\left[9 \left(3+2 x+3 x^2\right)^2\right]}{\sqrt{1+2 i \sqrt{2}}}+\frac{i \left(i+11 \sqrt{2}\right) \text{Log}\left[9 \left(3+2 x+3 x^2\right)^2\right]}{\sqrt{-1+2 i \sqrt{2}}}+\frac{1}{\sqrt{-1+2 i \sqrt{2}}} \left(1-11 i \sqrt{2}\right) \text{Log}\left[\left(3+2 x+3 x^2\right) \left(-7 i+4 \sqrt{2}+\left(-7 i+4 \sqrt{2}\right) x^2-2 i x \left(-3+4 \sqrt{-1+2 i \sqrt{2}} \sqrt{1+x^2}\right)\right)\right]+\frac{1}{\sqrt{1+2 i \sqrt{2}}} \left(-i+11 \sqrt{2}\right) \text{Log}\left[\left(3+2 x+3 x^2\right) \left(-11 i+4 \sqrt{2}+\left(-11 i+4 \sqrt{2}\right) x^2-6 i \sqrt{2+4 i \sqrt{2}} \sqrt{1+x^2}+2 i x \left(3+\sqrt{2+4 i \sqrt{2}} \sqrt{1+x^2}\right)\right)\right]\right)$$

Problem 264: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(-1+x^2\right) \sqrt{2 x+x^2}} \, dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{1}{2} \text{ArcTan}\left[\sqrt{2 x+x^2}\right]-\frac{\text{ArcTanh}\left[\frac{1+2 x}{\sqrt{3} \sqrt{2 x+x^2}}\right]}{2 \sqrt{3}}$$

Result (type 3, 113 leaves):

$$\frac{1}{6 \sqrt{x (2+x)}} \sqrt{x} \sqrt{2+x} \left(-6 \text{ArcTan}\left[\sqrt{\frac{x}{2+x}}\right]+\sqrt{3} \left(\text{Log}\left[1-\sqrt{x}\right]-\text{Log}\left[1+\sqrt{x}\right]+\text{Log}\left[2-\sqrt{x}+\sqrt{3} \sqrt{2+x}\right]-\text{Log}\left[2+\sqrt{x}+\sqrt{3} \sqrt{2+x}\right]\right)\right)$$

Problem 294: Result unnecessarily involves higher level functions.

$$\int \frac{(-1+3 x)^{4/3}}{x^2} \, dx$$

Optimal (type 3, 71 leaves, 6 steps):

$$12 (-1+3 x)^{1/3}-\frac{(-1+3 x)^{4/3}}{x}+4 \sqrt{3} \text{ArcTan}\left[\frac{1-2 (-1+3 x)^{1/3}}{\sqrt{3}}\right]+2 \text{Log}[x]-6 \text{Log}\left[1+(-1+3 x)^{1/3}\right]$$

Result (type 5, 59 leaves):

$$\frac{-1-6 x+27 x^2+2\times 3^{1/3} \left(3-\frac{1}{x}\right)^{2/3} x \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{3 x}\right]}{x (-1+3 x)^{2/3}}$$

Problem 296: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1-2 x^{1/3}\right)^{3/4}}{x} \, dx$$

Optimal (type 3, 48 leaves, 6 steps):

$$4 \left(1-2 x^{1/3}\right)^{3/4}+6 \text{ArcTan}\left[\left(1-2 x^{1/3}\right)^{1/4}\right]-6 \text{ArcTanh}\left[\left(1-2 x^{1/3}\right)^{1/4}\right]$$

Result (type 5, 62 leaves):

$$\frac{4-8x^{1/3}-6\times 2^{3/4}\left(2-\frac{1}{x^{1/3}}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2x^{1/3}}\right]}{\left(1-2x^{1/3}\right)^{1/4}}$$

Problem 298: Result unnecessarily involves higher level functions.

$$\int\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$$

Optimal (type 3, 193 leaves, 13 steps):

$$-\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}-\frac{5\left(-1+2\sqrt{x}\right)^{1/4}}{2\sqrt{x}}-\frac{5\,\text{ArcTan}\left[1-\sqrt{2}\left(-1+2\sqrt{x}\right)^{1/4}\right]}{2\sqrt{2}}+\frac{5\,\text{ArcTan}\left[1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{1/4}\right]}{2\sqrt{2}}-\frac{5\,\text{Log}\left[1-\sqrt{2}\left(-1+2\sqrt{x}\right)^{1/4}+\sqrt{-1+2\sqrt{x}}\right]}{4\sqrt{2}}+\frac{5\,\text{Log}\left[1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{1/4}+\sqrt{-1+2\sqrt{x}}\right]}{4\sqrt{2}}$$

Result (type 5, 72 leaves):

$$\frac{-6+39\sqrt{x}-54x-5\times 2^{1/4}\left(2-\frac{1}{\sqrt{x}}\right)^{3/4}x\,\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2\sqrt{x}}\right]}{6\left(-1+2\sqrt{x}\right)^{3/4}x}$$

Problem 301: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{3-2\left(-27+2x^7\right)^{1/3}}{3\sqrt{3}}\right]}{21\sqrt{3}}-\frac{\text{Log}[x]}{18}+\frac{1}{42}\text{Log}\left[3+\left(-27+2x^7\right)^{1/3}\right]$$

Result (type 5, 43 leaves):

$$-\frac{3\left(2-\frac{27}{x^7}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{27}{2x^7}\right]}{14\left(-54+4x^7\right)^{2/3}}$$

Problem 302: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1+x^7\right)^{2/3}}{x^8} \, dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$-\frac{\left(1+x^7\right)^{2/3}}{7 x^7}+\frac{2 \operatorname{ArcTan}\left[\frac{1+2 \left(1+x^7\right)^{1/3}}{\sqrt{3}}\right]}{7 \sqrt{3}}-\frac{\operatorname{Log}[x]}{3}+\frac{1}{7} \operatorname{Log}\left[1-\left(1+x^7\right)^{1/3}\right]$$

Result (type 5, 54 leaves):

$$-\frac{\left(1+x^7\right)^{2/3}}{7 x^7}-\frac{2 \left(1+\frac{1}{x^7}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{1}{x^7}\right]}{7 \left(1+x^7\right)^{1/3}}$$

Problem 303: Result unnecessarily involves higher level functions.

$$\int \frac{\left(3+4 x^4\right)^{1/4}}{x^2} \, dx$$

Optimal (type 3, 68 leaves, 5 steps):

$$-\frac{\left(3+4 x^4\right)^{1/4}}{x}-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{2} x}{\left(3+4 x^4\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{2} x}{\left(3+4 x^4\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 5, 46 leaves):

$$-\frac{\left(3+4 x^4\right)^{1/4}}{x}+\frac{4 x^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{4 x^4}{3}\right]}{3 \times 3^{3/4}}$$

Problem 304: Result unnecessarily involves higher level functions.

$$\int x^2 \left(3+4 x^4\right)^{5/4} \, dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{15}{32} x^3 \left(3+4 x^4\right)^{1/4}+\frac{1}{8} x^3 \left(3+4 x^4\right)^{5/4}-\frac{45 \operatorname{ArcTan}\left[\frac{\sqrt{2} x}{\left(3+4 x^4\right)^{1/4}}\right]}{128 \sqrt{2}}+\frac{45 \operatorname{ArcTanh}\left[\frac{\sqrt{2} x}{\left(3+4 x^4\right)^{1/4}}\right]}{128 \sqrt{2}}$$

Result (type 5, 51 leaves):

$$\frac{1}{32} x^3 \left(\left(3+4 x^4\right)^{1/4} \left(27+16 x^4\right) + 5 \times 3^{1/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{4 x^4}{3}\right]\right)$$

Problem 305: Result unnecessarily involves higher level functions.

$$\int x^6 \left(3+4 x^4\right)^{1/4} dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{3}{128} x^3 \left(3+4 x^4\right)^{1/4} + \frac{1}{8} x^7 \left(3+4 x^4\right)^{1/4} + \frac{27 \text{ArcTan}\left[\frac{\sqrt{2} x}{\left(3+4 x^4\right)^{1/4}}\right]}{512 \sqrt{2}} - \frac{27 \text{ArcTanh}\left[\frac{\sqrt{2} x}{\left(3+4 x^4\right)^{1/4}}\right]}{512 \sqrt{2}}$$

Result (type 5, 51 leaves):

$$\frac{1}{128} x^3 \left(\left(3+4 x^4\right)^{1/4} \left(3+16 x^4\right) - 3 \times 3^{1/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{4 x^4}{3}\right]\right)$$

Problem 306: Result unnecessarily involves higher level functions.

$$\int \left(x \left(1-x^2\right)\right)^{1/3} dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{1}{2} x \left(x \left(1-x^2\right)\right)^{1/3} + \frac{\text{ArcTan}\left[\frac{2 x-\left(x \left(1-x^2\right)\right)^{1/3}}{\sqrt{3} \left(x \left(1-x^2\right)\right)^{1/3}}\right]}{2 \sqrt{3}} + \frac{\text{Log}[x]}{12} - \frac{1}{4} \text{Log}\left[x+\left(x \left(1-x^2\right)\right)^{1/3}\right]$$

Result (type 5, 56 leaves):

$$\frac{x \left(x-x^3\right)^{1/3} \left(-2+2 x^2-\left(1-x^2\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, x^2\right]\right)}{4 \left(-1+x^2\right)}$$

Problem 311: Result more than twice size of optimal antiderivative.

$$\int \frac{-1+x^2}{x \sqrt{1+3 x^2+x^4}} dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$\text{ArcTanh}\left[\frac{1+x^2}{\sqrt{1+3 x^2+x^4}}\right]$$

Result (type 3, 59 leaves):

$$\frac{1}{2}\left(-\text{Log}\left[x^2\right]+\text{Log}\left[3+2 x^2+2 \sqrt{1+3 x^2+x^4}\right]+\text{Log}\left[2+3 x^2+2 \sqrt{1+3 x^2+x^4}\right]\right)$$

Problem 319: Unable to integrate problem.

$$\int \frac{1}{\left(3 x+3 x^2+x^3\right) \left(3+3 x+3 x^2+x^3\right)^{1/3}} \, dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\ 3^{1/3} (1+x)}{\left(2+(1+x)^3\right)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}-\frac{\text{Log}\left[1-(1+x)^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3} (1+x)-\left(2+(1+x)^3\right)^{1/3}\right]}{2\times 3^{1/3}}$$

Result (type 8, 34 leaves):

$$\int \frac{1}{\left(3 x+3 x^2+x^3\right) \left(3+3 x+3 x^2+x^3\right)^{1/3}} \, dx$$

Problem 320: Result unnecessarily involves higher level functions.

$$\int \frac{1-x^2}{\left(1+x^2\right) \sqrt{1+x^4}} \, dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\ x}{\sqrt{1+x^4}}\right]}{\sqrt{2}}$$

Result (type 4, 40 leaves):

$$(-1)^{1/4} \left(\text{EllipticF}\left[i \text{ ArcSinh}\left[(-1)^{1/4} x\right],-1\right]-2 \text{ EllipticPi}\left[-i,i \text{ ArcSinh}\left[(-1)^{1/4} x\right],-1\right]\right)$$

Problem 321: Result unnecessarily involves higher level functions.

$$\int \frac{1+x^2}{\left(1-x^2\right) \sqrt{1+x^4}} \, dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{\sqrt{2}}$$

Result (type 4, 36 leaves):

$$(-1)^{1/4}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]-2\,\text{EllipticPi}\left[i,\text{ArcSin}\left[(-1)^{3/4}\,x\right],-1\right]\right)$$

Problem 324: Unable to integrate problem.

$$\int\frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 3, 26 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,x}{\sqrt{1+x^2+x^4}}\right]}{\sqrt{3}}$$

Result (type 8, 29 leaves):

$$\int\frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

Problem 325: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]$$

Result (type 4, 94 leaves):

$$-\frac{1}{\sqrt{1+x^2+x^4}}(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}\,x\right],(-1)^{2/3}\right]+2\,\text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}\,x\right],(-1)^{2/3}\right]\right)$$

Problem 327: Result unnecessarily involves higher level functions and more than twice size of optimal

antiderivative.

$$\int \frac{1-x^2}{\left(1+2 a x+x^2\right) \sqrt{1+2 a x+2 b x^2+2 a x^3+x^4}} \, dx$$

Optimal (type 3, 74 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{a+2\left(1+a^2-b\right) x+a x^2}{\sqrt{2} \sqrt{1-b} \sqrt{1+2 a x+2 b x^2+2 a x^3+x^4}}\right]}{\sqrt{2} \sqrt{1-b}}$$

Result (type 4, 17 955 leaves):

```
(2 a (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])^2
  (EllipticF[ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
          Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
        (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
     -(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
        (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
       ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &,
          2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))] (-a + √(-1 + a^2) - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) -
    EllipticPi[((a - √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
          Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((a - √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
        (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])),
     ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
          Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
        (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
     -(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
        (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
       ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
        (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
    (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]))
  √(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])))
  √(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
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Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))/((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
√(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))/((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))/
($\sqrt{1 + 2 b x^2 + x^4 + 2 a (x + x^3)}$
(a - $\sqrt{-1 + a^2}$ +
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
(-a + $\sqrt{-1 + a^2}$ - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) +
((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])^2
(EllipticF[ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))/((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
-(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))/
((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &,
2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))] (-a + $\sqrt{-1 + a^2}$ - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) -
EllipticPi[((a - $\sqrt{-1 + a^2}$ + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))/((a - $\sqrt{-1 + a^2}$ + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])),
ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))/((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
-(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))/

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((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
  (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))))]
 (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]))
√(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])))
√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
     Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
√(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / (√(-1 + a^2)
√(1 + 2 b x^2 + x^4 + 2 a (x + x^3))
(a - √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
(-a + √(-1 + a^2) - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
  Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) -
(a^2 (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])^2
 (EllipticF[ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
         Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
       (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
   -(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
     ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &,
         2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))] (-a + √(-1 + a^2) - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) -
  EllipticPi[((a - √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
        Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((a - √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])),
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ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
        Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
  - (((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
    ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
 (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]))

√(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
    (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
    (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])))
√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
      Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
    (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
√(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
    (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
    (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / (√(-1 + a^2)
√(1 + 2 b x^2 + x^4 + 2 a (x + x^3))
(a - √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
(-a + √(-1 + a^2) - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
  Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) -
(√(-1 + a^2) (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]))^2
(EllipticF[ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
        Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
  - (((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
    ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &,
        2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))] (-a + √(-1 + a^2) - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) -
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EllipticPi[((a - Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
      Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((a - Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
    (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])),
  ArcSin[Sqrt[(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
          Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
        (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]],
  -(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
    ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
  (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]))
 Sqrt[(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])))]
 Sqrt[(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
        Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
 Sqrt[(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
 (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
(Sqrt[1 + 2 b x^2 + x^4 + 2 a (x + x^3)]
 (a - Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
 (-a + Sqrt[-1 + a^2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
 (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
   Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
 (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) + (2
 a
 (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])^2
 (EllipticF[ArcSin[Sqrt[(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
          Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
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(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
 -(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
     (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
   ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &,
       2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))] (-a - Sqrt[-1 + a^2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) -
  EllipticPi[((a + Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
        Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((a + Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])),
   ArcSin[Sqrt[(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
          Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
        (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
   -(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
       (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
     ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
       (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
  (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]))
 Sqrt[(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])))]
 Sqrt[(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
        Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
 Sqrt[(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
 (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
(Sqrt[1 + 2 b x^2 + x^4 + 2 a (x + x^3)]
  (a + Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
  (-a - Sqrt[-1 + a^2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
  (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
    Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
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(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) -
((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])^2
(EllipticF[ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
-(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &,
2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))] (-a - √(-1 + a^2) - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) -
EllipticPi[((a + √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((a + √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])),
ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
-(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]))
√(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])))
√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
√(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / (√(-1 + a^2)
√(1 + 2 b x^2 + x^4 + 2 a (x + x^3))
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(a + √(-1 + a²) + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1])
(-a - √(-1 + a²) - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])
(-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] +
   Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])
(Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])) +
(a² (x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])²
 (EllipticF[ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1]) (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2] -
         Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])
       (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])))],
    -(((Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 3])
       (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])) /
     ((-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 3]) (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &,
         2] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])))] (-a - √(-1 + a²) - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1]) -
   EllipticPi[((a + √(-1 + a²) + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2]) (-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] +
         Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])) / ((a + √(-1 + a²) + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1])
       (-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2] + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])),
    ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1]) (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2] -
         Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])
       (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])))],
    -(((Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 3])
       (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])) /
     ((-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 3])
       (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])))]
   (-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2]))
 √(((-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])
     (x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])
     (-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 3])))
 √(((x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1]) (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2] -
        Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])
     (Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] - Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 4])))
 √(((-Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 1] + Root[1 + 2 a #1 + 2 b #1² + 2 a #1³ + #1⁴ &, 2])
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          (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
          (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / (Sqrt[-1 + a^2]
      Sqrt[1 + 2 b x^2 + x^4 + 2 a (x + x^3)]
      (a + Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
      (-a - Sqrt[-1 + a^2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
        Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
      (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) +
    (Sqrt[-1 + a^2] (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])^2
      (EllipticF[ArcSin[Sqrt[(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
                  Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
                (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]],
          -(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
               (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
             ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &,
                  2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))] (-a - Sqrt[-1 + a^2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) -
        EllipticPi[((a + Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
                Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((a + Sqrt[-1 + a^2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
              (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])),
          ArcSin[Sqrt[(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
                  Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
                (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]],
          -(((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
               (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
             ((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
               (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))]
        (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2]))
      Sqrt[(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
          (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
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(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])))
√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] -
    Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
  (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
√(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
  (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
  (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
(-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
(√(1 + 2 b x^2 + x^4 + 2 a (x + x^3))
 (a + √(-1 + a^2) + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1])
 (-a - √(-1 + a^2) - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
 (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] +
   Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
 (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) -
(2 EllipticF[ArcSin[√(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] +
      Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
     (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))],
  ((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
    (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) /
   ((Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])
    (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4]))]
 (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])^2
 √(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 3])))
 (Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])
 √(((-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))
 √(((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1]) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] +
     Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])) / ((x - Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
   (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 4])))) /
(√(1 + 2 b x^2 + x^4 + 2 a (x + x^3)) (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 1] + Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2])
 (-Root[1 + 2 a #1 + 2 b #1^2 + 2 a #1^3 + #1^4 &, 2] +
```

$\text{Root}\left[1+2\,a\,\#1+2\,b\,\#1^2+2\,a\,\#1^3+\#1^4\,\&,\,4\right]\Big)\Big)$

Problem 338: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[x]^7\,dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{5}{16}\,\text{ArcTanh}[\text{Cos}[x]]-\frac{5}{16}\,\text{Cot}[x]\,\text{Csc}[x]-\frac{5}{24}\,\text{Cot}[x]\,\text{Csc}[x]^3-\frac{1}{6}\,\text{Cot}[x]\,\text{Csc}[x]^5$$

Result (type 3, 95 leaves):

$$-\frac{5}{64}\,\text{Csc}\left[\frac{x}{2}\right]^2-\frac{1}{64}\,\text{Csc}\left[\frac{x}{2}\right]^4-\frac{1}{384}\,\text{Csc}\left[\frac{x}{2}\right]^6-\frac{5}{16}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\frac{5}{16}\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{5}{64}\,\text{Sec}\left[\frac{x}{2}\right]^2+\frac{1}{64}\,\text{Sec}\left[\frac{x}{2}\right]^4+\frac{1}{384}\,\text{Sec}\left[\frac{x}{2}\right]^6$$

Problem 355: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]^3\,\text{Csc}[x]\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\text{Csc}[x]-\frac{\text{Csc}[x]^3}{3}$$

Result (type 3, 57 leaves):

$$\frac{5}{12}\,\text{Cot}\left[\frac{x}{2}\right]-\frac{1}{24}\,\text{Cot}\left[\frac{x}{2}\right]\,\text{Csc}\left[\frac{x}{2}\right]^2+\frac{5}{12}\,\text{Tan}\left[\frac{x}{2}\right]-\frac{1}{24}\,\text{Sec}\left[\frac{x}{2}\right]^2\,\text{Tan}\left[\frac{x}{2}\right]$$

Problem 357: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]^2\,\text{Csc}[x]^3\,dx$$

Optimal (type 3, 26 leaves, 3 steps):

$$\frac{1}{8}\,\text{ArcTanh}[\text{Cos}[x]]+\frac{1}{8}\,\text{Cot}[x]\,\text{Csc}[x]-\frac{1}{4}\,\text{Cot}[x]\,\text{Csc}[x]^3$$

Result (type 3, 71 leaves):

$$\frac{1}{32}\,\text{Csc}\left[\frac{x}{2}\right]^2-\frac{1}{64}\,\text{Csc}\left[\frac{x}{2}\right]^4+\frac{1}{8}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]-\frac{1}{8}\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]-\frac{1}{32}\,\text{Sec}\left[\frac{x}{2}\right]^2+\frac{1}{64}\,\text{Sec}\left[\frac{x}{2}\right]^4$$

Problem 361: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]^4 \text{Csc}[x]^3 \, dx$$

Optimal (type 3, 38 leaves, 4 steps):

$$-\frac{1}{16} \text{ArcTanh}[\text{Cos}[x]] - \frac{1}{16} \text{Cot}[x] \text{Csc}[x] + \frac{1}{8} \text{Cot}[x] \text{Csc}[x]^3 - \frac{1}{6} \text{Cot}[x]^3 \text{Csc}[x]^3$$

Result (type 3, 95 leaves):

$$-\frac{1}{64} \text{Csc}\left[\frac{x}{2}\right]^2 + \frac{1}{64} \text{Csc}\left[\frac{x}{2}\right]^4 - \frac{1}{384} \text{Csc}\left[\frac{x}{2}\right]^6 - \frac{1}{16} \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \frac{1}{16} \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] + \frac{1}{64} \text{Sec}\left[\frac{x}{2}\right]^2 - \frac{1}{64} \text{Sec}\left[\frac{x}{2}\right]^4 + \frac{1}{384} \text{Sec}\left[\frac{x}{2}\right]^6$$

Problem 367: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[4 x] \text{Sec}[x] \, dx$$

Optimal (type 3, 12 leaves, 4 steps):

$$\text{ArcTanh}[\text{Sin}[x]] - \frac{8 \text{Sin}[x]^3}{3}$$

Result (type 3, 45 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] - 2 \text{Sin}[x] + \frac{2}{3} \text{Sin}[3 x]$$

Problem 369: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[4 x] \text{Sec}[x]^5 \, dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$\frac{35}{8} \text{ArcTanh}[\text{Sin}[x]] - \frac{29}{8} \text{Sec}[x] \text{Tan}[x] + \frac{1}{4} \text{Sec}[x]^3 \text{Tan}[x]$$

Result (type 3, 58 leaves):

$$\frac{1}{16} \left(-70 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + 70 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] - \frac{1}{2} \text{Sec}[x]^4 (21 \text{Sin}[x] + 29 \text{Sin}[3 x])\right)$$

Problem 383: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[x]^2\,\text{Sec}[3x]\,dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\frac{1}{2}\,\text{ArcTanh}[2\,\text{Sin}[x]]$$

Result (type 3, 23 leaves):

$$-\frac{1}{4}\,\text{Log}[1-2\,\text{Sin}[x]]+\frac{1}{4}\,\text{Log}[1+2\,\text{Sin}[x]]$$

Problem 384: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Sec}[2x]\,\text{Sin}[x]\,dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\sqrt{2}\,\text{Cos}[x]\right]}{\sqrt{2}}$$

Result (type 3, 174 leaves):

$$\frac{1}{4\sqrt{2}}\left(2\,i\,\text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]-\left(-1+\sqrt{2}\right)\text{Sin}\left[\frac{x}{2}\right]}{\left(1+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}\right]-2\,i\,\text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]-\left(1+\sqrt{2}\right)\text{Sin}\left[\frac{x}{2}\right]}{\left(-1+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}\right]+\right.$$
$$\left.4\,\text{ArcTanh}\left[\sqrt{2}+\text{Tan}\left[\frac{x}{2}\right]\right]-\text{Log}\left[2-\sqrt{2}\,\text{Cos}[x]-\sqrt{2}\,\text{Sin}[x]\right]+\text{Log}\left[2+\sqrt{2}\,\text{Cos}[x]-\sqrt{2}\,\text{Sin}[x]\right]\right)$$

Problem 388: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Csc}[4x]\,\text{Sin}[x]\,dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4}\,\text{ArcTanh}[\text{Sin}[x]]+\frac{\text{ArcTanh}\left[\sqrt{2}\,\text{Sin}[x]\right]}{2\sqrt{2}}$$

Result (type 3, 218 leaves):

$$\frac{1}{8\sqrt{2}}\left(-2\,i\,\text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]-\left(-1+\sqrt{2}\right)\text{Sin}\left[\frac{x}{2}\right]}{\left(1+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}\right]-2\,i\,\text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]-\left(1+\sqrt{2}\right)\text{Sin}\left[\frac{x}{2}\right]}{\left(-1+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}\right]+2\sqrt{2}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]-2\sqrt{2}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+2\,\text{Log}\left[\sqrt{2}+2\,\text{Sin}[x]\right]-\text{Log}\left[2-\sqrt{2}\,\text{Cos}[x]-\sqrt{2}\,\text{Sin}[x]\right]-\text{Log}\left[2+\sqrt{2}\,\text{Cos}[x]-\sqrt{2}\,\text{Sin}[x]\right]\right)$$

Problem 389: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Csc}[4\,x]\,\text{Sin}[x]^3\,dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4}\text{ArcTanh}[\text{Sin}[x]]+\frac{\text{ArcTanh}\left[\sqrt{2}\,\text{Sin}[x]\right]}{4\sqrt{2}}$$

Result (type 3, 218 leaves):

$$\frac{1}{16\sqrt{2}}\left(-2\,i\,\text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]-\left(-1+\sqrt{2}\right)\text{Sin}\left[\frac{x}{2}\right]}{\left(1+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}\right]-2\,i\,\text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right]-\left(1+\sqrt{2}\right)\text{Sin}\left[\frac{x}{2}\right]}{\left(-1+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}\right]+4\sqrt{2}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]-4\sqrt{2}\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]+2\,\text{Log}\left[\sqrt{2}+2\,\text{Sin}[x]\right]-\text{Log}\left[2-\sqrt{2}\,\text{Cos}[x]-\sqrt{2}\,\text{Sin}[x]\right]-\text{Log}\left[2+\sqrt{2}\,\text{Cos}[x]-\sqrt{2}\,\text{Sin}[x]\right]\right)$$

Problem 398: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\text{Tan}[5\,x]^{1/3}}\,dx$$

Optimal (type 3, 57 leaves, 9 steps):

$$-\frac{1}{10}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2\,\text{Tan}[5\,x]^{2/3}}{\sqrt{3}}\right]+\frac{3}{20}\text{Log}\left[1+\text{Tan}[5\,x]^{2/3}\right]-\frac{1}{20}\text{Log}\left[1+\text{Tan}[5\,x]^2\right]$$

Result (type 3, 121 leaves):

$$\frac{1}{20}\left(-2\sqrt{3}\,\text{ArcTan}\left[\sqrt{3}-2\,\text{Tan}[5\,x]^{1/3}\right]-2\sqrt{3}\,\text{ArcTan}\left[\sqrt{3}+2\,\text{Tan}[5\,x]^{1/3}\right]+2\,\text{Log}\left[1+\text{Tan}[5\,x]^{2/3}\right]-\text{Log}\left[1-\sqrt{3}\,\text{Tan}[5\,x]^{1/3}+\text{Tan}[5\,x]^{2/3}\right]-\text{Log}\left[1+\sqrt{3}\,\text{Tan}[5\,x]^{1/3}+\text{Tan}[5\,x]^{2/3}\right]\right)$$

Problem 399: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(4+3\,\text{Tan}[2\,x])^{3/2}}\,dx$$

Optimal (type 3, 87 leaves, 6 steps):

$$-\frac{9\,\text{ArcTan}\left[\frac{1-3\,\text{Tan}[2\,x]}{\sqrt{2}\,\sqrt{4+3\,\text{Tan}[2\,x]}}\right]}{250\,\sqrt{2}}+\frac{13\,\text{ArcTanh}\left[\frac{3+\text{Tan}[2\,x]}{\sqrt{2}\,\sqrt{4+3\,\text{Tan}[2\,x]}}\right]}{250\,\sqrt{2}}-\frac{3}{25\,\sqrt{4+3\,\text{Tan}[2\,x]}}$$

Result (type 3, 83 leaves):

$$\frac{(24-7\,i)\,\sqrt{4-3\,i}\,\text{ArcTanh}\left[\frac{\sqrt{4+3\,\text{Tan}[2\,x]}}{\sqrt{4-3\,i}}\right]+(24+7\,i)\,\sqrt{4+3\,i}\,\text{ArcTanh}\left[\frac{\sqrt{4+3\,\text{Tan}[2\,x]}}{\sqrt{4+3\,i}}\right]-\frac{150}{\sqrt{4+3\,\text{Tan}[2\,x]}}}{1250}$$

Problem 411: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x]^3\,(\text{Cos}[2\,x]-3\,\text{Tan}[x])}{(\text{Sin}[x]^2-\text{Sin}[2\,x])\,\text{Sin}[2\,x]^{5/2}}\,dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{33}{32}\,\text{ArcTanh}\left[\frac{1}{2}\,\text{Sec}[x]\,\sqrt{\text{Sin}[2\,x]}\right]-\frac{9\,\text{Cos}[x]}{16\,\sqrt{\text{Sin}[2\,x]}}-\frac{5\,\text{Cos}[x]\,\text{Cot}[x]}{24\,\sqrt{\text{Sin}[2\,x]}}+\frac{\text{Cos}[x]\,\text{Cot}[x]^2}{20\,\sqrt{\text{Sin}[2\,x]}}$$

Result (type 4, 150 leaves):

$$\left(\text{Cos}[x]\sqrt{\text{Sin}[2x]}\left(\frac{1}{15}\text{Csc}[x]\left(-147-50\,\text{Cot}[x]+12\,\text{Csc}[x]^2\right)-\right.\right.$$

$$33\sqrt{\frac{\text{Cos}[x]}{-2+2\,\text{Cos}[x]}}\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{x}{2}\right]}}\right],-1\right]+\text{EllipticPi}\left[-\frac{2}{-1+\sqrt{5}},-\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{x}{2}\right]}}\right],-1\right]+\right.$$

$$\left.\left.\left.\text{EllipticPi}\left[\frac{1}{2}\left(-1+\sqrt{5}\right),-\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{x}{2}\right]}}\right],-1\right]\right)\text{Sec}[x]\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right)\left(\text{Cos}[2x]-3\,\text{Tan}[x]\right)\right)\Big/\left(16\left(\text{Cos}[x]+\text{Cos}[3x]-6\,\text{Sin}[x]\right)\right)$$

Problem 416: Result unnecessarily involves higher level functions.

$$\int\frac{\text{Cos}[2x]-\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\sqrt{2}\,\text{Log}\left[\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sec}[x]\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\right]-$$

$$\frac{\text{ArcSin}[\text{Cos}[x]-\text{Sin}[x]]\,\text{Cos}[x]\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{ArcTanh}[\text{Sin}[x]]\,\text{Cos}[x]\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{Sin}[2x]}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}$$

Result (type 5, 105 leaves):

$$\left(-4\,\text{Cos}[x]^3\,\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\text{Cos}[x]^2\right]\text{Sin}[x]-\right.$$

$$\left.3\,\text{Cos}[x]\left(\text{Sin}[x]^2\right)^{1/4}\left(2\,\text{Sin}[x]+\left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]\right)\sqrt{\text{Sin}[2x]}\right)\right)\Big/\left(3\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\left(\text{Sin}[x]^2\right)^{1/4}\right)$$

Problem 417: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\text{Cos}[x]\ \text{Sin}[x]^3}-2\ \text{Sin}[2\ x]}{-\sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}+\sqrt{\text{Tan}[x]}}\,dx$$

Optimal (type 3, 364 leaves, 66 steps):

$$-2\sqrt{2}\ \text{ArcCoth}\left[\frac{\text{Cos}[x]\ (\text{Cos}[x]+\text{Sin}[x])}{\sqrt{2}\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}}\right]+2^{1/4}\ \text{ArcCoth}\left[\frac{\text{Cos}[x]\ \left(\sqrt{2}\ \text{Cos}[x]+\text{Sin}[x]\right)}{2^{3/4}\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}}\right]-2^{1/4}\ \text{ArcCoth}\left[\frac{\sqrt{2}+\text{Tan}[x]}{2^{3/4}\ \sqrt{\text{Tan}[x]}}\right]-$$
$$2\sqrt{2}\ \text{ArcTan}\left[\frac{\text{Cos}[x]\ (\text{Cos}[x]-\text{Sin}[x])}{\sqrt{2}\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}}\right]+2^{1/4}\ \text{ArcTan}\left[\frac{\text{Cos}[x]\ \left(\sqrt{2}\ \text{Cos}[x]-\text{Sin}[x]\right)}{2^{3/4}\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}}\right]-2^{1/4}\ \text{ArcTan}\left[\frac{\sqrt{2}-\text{Tan}[x]}{2^{3/4}\ \sqrt{\text{Tan}[x]}}\right]+$$
$$4\ \text{Csc}[x]\ \text{Sec}[x]\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}+\frac{1}{4}\ \text{Csc}[x]^2\ \text{Log}\left[1+\text{Cos}[x]^2\right]\ \text{Sec}[x]^2\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}\ \sqrt{\text{Cos}[x]\ \text{Sin}[x]^3}+$$
$$\frac{1}{2}\ \text{Csc}[x]^2\ \text{Log}[\text{Sin}[x]]\ \text{Sec}[x]^2\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}\ \sqrt{\text{Cos}[x]\ \text{Sin}[x]^3}+\frac{4}{\sqrt{\text{Tan}[x]}}-$$
$$\frac{1}{4}\ \text{Csc}[x]^2\ \text{Log}\left[1+\text{Cos}[x]^2\right]\ \sqrt{\text{Cos}[x]\ \text{Sin}[x]^3}\ \sqrt{\text{Tan}[x]}+\frac{1}{2}\ \text{Csc}[x]^2\ \text{Log}[\text{Sin}[x]]\ \sqrt{\text{Cos}[x]\ \text{Sin}[x]^3}\ \sqrt{\text{Tan}[x]}$$

Result (type 5, 2057 leaves):

$$-\frac{\text{Cos}[x]\ \text{Csc}\left[\frac{x}{2}\right]\ \left(4\ \text{Log}\left[\text{Sec}\left[\frac{x}{2}\right]^2\right]-2\ \text{Log}\left[\text{Tan}\left[\frac{x}{2}\right]\right]-\text{Log}\left[1+\text{Tan}\left[\frac{x}{2}\right]^4\right]\right)\ \text{Sec}\left[\frac{x}{2}\right]\ \sqrt{\text{Cos}[x]\ \text{Sin}[x]^3}}{8\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}}+$$
$$\left((1+i)\ \left((4+4\,i)\ \text{EllipticPi}\left[-i,\ -\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],\ -1\right]-(4+4\,i)\ \text{EllipticPi}\left[i,\ -\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],\ -1\right]+\right.\right.$$
$$(-1)^{1/4}\ \left(-\text{EllipticPi}\left[-(-1)^{1/4},\ -\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],\ -1\right]+\text{EllipticPi}\left[(-1)^{1/4},\ -\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],\ -1\right]-\right.$$
$$\left.\left.\text{EllipticPi}\left[-(-1)^{3/4},\ -\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],\ -1\right]+\text{EllipticPi}\left[(-1)^{3/4},\ -\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],\ -1\right]\right)\right)$$
$$\left.\text{Sec}\left[\frac{x}{2}\right]^4\ \sqrt{\text{Cos}[x]^3\ \text{Sin}[x]}\ \left(\frac{2\sqrt{2}\ \text{Sec}[x]^2\ \sqrt{2\ \text{Sin}[2\ x]+\text{Sin}[4\ x]}}{3+\text{Cos}[2\ x]}+\frac{\sqrt{2}\ \text{Cos}[2\ x]\ \text{Sec}[x]^2\ \sqrt{2\ \text{Sin}[2\ x]+\text{Sin}[4\ x]}}{3+\text{Cos}[2\ x]}\right)\right)\Big/$$

$$\left(\sqrt{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}\,\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\,\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)\left(-\frac{1}{\sqrt{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}\,\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}\right.\right.$$

$$(1+\mathrm{i})\left((4+4\,\mathrm{i})\,\text{EllipticPi}\left[-\mathrm{i},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-(4+4\,\mathrm{i})\,\text{EllipticPi}\left[\mathrm{i},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\right.$$

$$(-1)^{1/4}\left(-\text{EllipticPi}\left[-(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-\right.$$

$$\left.\left.\text{EllipticPi}\left[-(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]\right)\right)$$

$$\text{Sec}\left[\frac{x}{2}\right]^6\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\,\sqrt{\text{Tan}\left[\frac{x}{2}\right]}-\frac{1}{\sqrt{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}\,\text{Tan}\left[\frac{x}{2}\right]^{3/2}\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

$$\left(\frac{1}{4}+\frac{\mathrm{i}}{4}\right)\left((4+4\,\mathrm{i})\,\text{EllipticPi}\left[-\mathrm{i},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-(4+4\,\mathrm{i})\,\text{EllipticPi}\left[\mathrm{i},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\right.$$

$$(-1)^{1/4}\left(-\text{EllipticPi}\left[-(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-\right.$$

$$\left.\left.\text{EllipticPi}\left[-(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]\right)\right)$$

$$\text{Sec}\left[\frac{x}{2}\right]^6\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}+\frac{1}{\sqrt{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\,\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

$$\left(\frac{1}{2}+\frac{\mathrm{i}}{2}\right)\left((4+4\,\mathrm{i})\,\text{EllipticPi}\left[-\mathrm{i},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-(4+4\,\mathrm{i})\,\text{EllipticPi}\left[\mathrm{i},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\right.$$

$$(-1)^{1/4}\left(-\text{EllipticPi}\left[-(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-\right.$$

$$\left.\left.\text{EllipticPi}\left[-(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]\right)\right)$$

$$\text{Sec}\left[\frac{x}{2}\right]^4\left(\text{Cos}[x]^4-3\,\text{Cos}[x]^2\,\text{Sin}[x]^2\right)+\frac{1}{\sqrt{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

$$(2+2\,i)\left((4+4\,i)\,\text{EllipticPi}\left[-i,-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-(4+4\,i)\,\text{EllipticPi}\left[i,-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+(-1)^{1/4}\left(-\text{EllipticPi}\left[-(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-\text{EllipticPi}\left[-(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]\right)\right)$$

$$\text{Sec}\left[\frac{x}{2}\right]^4\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\,\sqrt{\text{Tan}\left[\frac{x}{2}\right]}-\frac{1}{\left(\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2\right)^{3/2}\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left((4+4\,i)\,\text{EllipticPi}\left[-i,-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-(4+4\,i)\,\text{EllipticPi}\left[i,-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+(-1)^{1/4}\left(-\text{EllipticPi}\left[-(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{1/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]-\text{EllipticPi}\left[-(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]+\text{EllipticPi}\left[(-1)^{3/4},-\text{ArcSin}\left[\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\right],-1\right]\right)\right)\text{Sec}\left[\frac{x}{2}\right]^4$$

$$\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\left(-\text{Sec}\left[\frac{x}{2}\right]^2\text{Sin}[x]+\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]\right)+\frac{1}{\sqrt{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

$$(1+i)\,\text{Sec}\left[\frac{x}{2}\right]^4\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\left(\frac{(1+i)\,\text{Sec}\left[\frac{x}{2}\right]^2}{\sqrt{1-\text{Tan}\left[\frac{x}{2}\right]}\left(1-i\,\text{Tan}\left[\frac{x}{2}\right]\right)\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\sqrt{1+\text{Tan}\left[\frac{x}{2}\right]}}-\right.$$

$$\frac{(1+i)\,\text{Sec}\left[\frac{x}{2}\right]^2}{\sqrt{1-\text{Tan}\left[\frac{x}{2}\right]}\left(1+i\,\text{Tan}\left[\frac{x}{2}\right]\right)\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\sqrt{1+\text{Tan}\left[\frac{x}{2}\right]}}+(-1)^{1/4}\left(-\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{4\sqrt{1-\text{Tan}\left[\frac{x}{2}\right]}\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\sqrt{1+\text{Tan}\left[\frac{x}{2}\right]}\left(1-(-1)^{1/4}\,\text{Tan}\left[\frac{x}{2}\right]\right)}+\right.$$

$$\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{4\sqrt{1-\text{Tan}\left[\frac{x}{2}\right]}\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\sqrt{1+\text{Tan}\left[\frac{x}{2}\right]}\left(1+(-1)^{1/4}\,\text{Tan}\left[\frac{x}{2}\right]\right)}-\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{4\sqrt{1-\text{Tan}\left[\frac{x}{2}\right]}\sqrt{\text{Tan}\left[\frac{x}{2}\right]}\sqrt{1+\text{Tan}\left[\frac{x}{2}\right]}\left(1-(-1)^{3/4}\,\text{Tan}\left[\frac{x}{2}\right]\right)}+$$

$$\left.\left.\left.\left.\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{4\sqrt{1-\text{Tan}\left[\frac{x}{2}\right]}\ \sqrt{\text{Tan}\left[\frac{x}{2}\right]}\ \sqrt{1+\text{Tan}\left[\frac{x}{2}\right]}\ \left(1+(-1)^{3/4}\,\text{Tan}\left[\frac{x}{2}\right]\right)}\right)\right)\right)\right)+$$

$$\frac{4}{\sqrt{\text{Tan}[x]}}-\frac{2\left(\text{Cos}[x]^2\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{\text{Sin}[x]^2}{2\left(1-\frac{\text{Sin}[x]^2}{2}\right)}\right]\left(2-\text{Sin}[x]^2\right)\text{Tan}[x]^{3/2}}{3\left(1-\frac{\text{Sin}[x]^2}{2}\right)^{3/4}\left(-2+\text{Sin}[x]^2\right)}+$$

$$\sqrt{2\,\text{Sin}[2x]+\text{Sin}[4x]}$$
$$\left(\sqrt{2}\right.$$
$$\left.\text{Cot}[x]+\sqrt{2}\ \text{Tan}[x]\right)+$$
$$\left(\text{Csc}[x]^2\left(4\,\text{Log}\left[\sqrt{\text{Tan}[x]}\right]-\text{Log}\left[2+\text{Tan}[x]^2\right]\right)\right.$$
$$\text{Sec}[x]^2$$
$$\sqrt{2\,\text{Sin}[2x]-\text{Sin}[4x]}$$
$$\sqrt{\text{Tan}[x]}$$
$$\left.\left(2+\text{Tan}[x]^2\right)\right)\Big/\left(4\right.$$
$$\sqrt{2}$$
$$\left(3+\text{Cos}[2x]\right)$$
$$\left.\left(1+\text{Tan}[x]^2\right)^2\right)$$

Problem 424: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Sin}[5x]}{\left(5\,\text{Cos}[x]^2+9\,\text{Sin}[x]^2\right)^{5/2}}\,dx$$

Optimal (type 3, 48 leaves, 4 steps):

$$-\frac{1}{2}\,\text{ArcSin}\left[\frac{2\,\text{Cos}[x]}{3}\right]-\frac{55\,\text{Cos}[x]}{27\left(9-4\,\text{Cos}[x]^2\right)^{3/2}}+\frac{295\,\text{Cos}[x]}{243\sqrt{9-4\,\text{Cos}[x]^2}}$$

Result (type 3, 63 leaves):

$$\frac{2550\,\text{Cos}[x]-590\,\text{Cos}[3x]+243\,i\left(7-2\,\text{Cos}[2x]\right)^{3/2}\text{Log}\left[2\,i\,\text{Cos}[x]+\sqrt{7-2\,\text{Cos}[2x]}\right]}{486\left(7-2\,\text{Cos}[2x]\right)^{3/2}}$$

Problem 426: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[x]^2 \left(-2\,\text{Cos}[x]^3 \left(-1+\text{Sin}[x]\right)+\text{Cos}[2\,x]\,\text{Sin}[x]\right)}{\sqrt{-5+\text{Sin}[x]^2}}\,dx$$

Optimal (type 3, 111 leaves, 18 steps):

$$2\,\text{ArcTan}\left[\frac{\text{Cos}[x]}{\sqrt{-5+\text{Sin}[x]^2}}\right]-\frac{\text{ArcTan}\left[\frac{\sqrt{5}\,\text{Cos}[x]}{\sqrt{-5+\text{Sin}[x]^2}}\right]}{\sqrt{5}}-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-5+\text{Sin}[x]^2}}{\sqrt{5}}\right]}{\sqrt{5}}-2\,\text{ArcTanh}\left[\frac{\text{Sin}[x]}{\sqrt{-5+\text{Sin}[x]^2}}\right]+2\sqrt{-5+\text{Sin}[x]^2}+\frac{2}{5}\,\text{Csc}[x]\sqrt{-5+\text{Sin}[x]^2}$$

Result (type 4, 338 leaves):

$$\frac{1}{25\sqrt{2}\,\sqrt{-9-\text{Cos}[2\,x]}}\left(\left(16-32\,i\right)\sqrt{5}\,\text{Cos}\left[\frac{x}{2}\right]^2\sqrt{\frac{\left(1+2\,i\right)\left(-2\,i+\text{Cos}[x]\right)}{1+\text{Cos}[x]}}\sqrt{\frac{\left(1-2\,i\right)\left(2\,i+\text{Cos}[x]\right)}{1+\text{Cos}[x]}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+2\,i\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{5}}\right],-\frac{7}{25}+\frac{24\,i}{25}\right]-\left(32-64\,i\right)\sqrt{5}\,\text{Cos}\left[\frac{x}{2}\right]^2\sqrt{\frac{\left(1+2\,i\right)\left(-2\,i+\text{Cos}[x]\right)}{1+\text{Cos}[x]}}\sqrt{\frac{\left(1-2\,i\right)\left(2\,i+\text{Cos}[x]\right)}{1+\text{Cos}[x]}}\,\text{EllipticPi}\left[\frac{3}{5}+\frac{4\,i}{5},\text{ArcSin}\left[\frac{\left(1+2\,i\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{5}}\right],-\frac{7}{25}+\frac{24\,i}{25}\right]-5\left(85+\sqrt{10}\,\text{ArcTan}\left[\frac{\sqrt{10}\,\text{Cos}[x]}{\sqrt{-9-\text{Cos}[2\,x]}}\right]\sqrt{-9-\text{Cos}[2\,x]}+2\sqrt{10}\,\text{ArcTan}\left[\frac{\sqrt{-9-\text{Cos}[2\,x]}}{\sqrt{10}}\right]\sqrt{-9-\text{Cos}[2\,x]}+18\,\text{Csc}[x]+2\,\text{Cos}[2\,x]\,\text{Csc}[x]+10\,i\sqrt{2}\,\sqrt{-9-\text{Cos}[2\,x]}\,\text{Log}\left[i\sqrt{2}\,\text{Cos}[x]+\sqrt{-9-\text{Cos}[2\,x]}\right]+5\,\text{Csc}[x]\,\text{Sin}[3\,x]\right)\right)$$

Problem 427: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cos}[3\,x]}{-\sqrt{-1+8\,\text{Cos}[x]^2}+\sqrt{3\,\text{Cos}[x]^2-\text{Sin}[x]^2}}\,dx$$

Optimal (type 3, 112 leaves, 27 steps):

$$\frac{5\,\text{ArcSin}\left[2\sqrt{\frac{2}{7}}\,\text{Sin}[x]\right]}{4\sqrt{2}}+\frac{3}{4}\text{ArcSin}\left[\frac{2\,\text{Sin}[x]}{\sqrt{3}}\right]-\frac{3}{4}\text{ArcTan}\left[\frac{\text{Sin}[x]}{\sqrt{-1+4\,\text{Cos}[x]^2}}\right]-$$
$$\frac{3}{4}\text{ArcTan}\left[\frac{\text{Sin}[x]}{\sqrt{-1+8\,\text{Cos}[x]^2}}\right]-\frac{1}{2}\sqrt{-1+4\,\text{Cos}[x]^2}\,\text{Sin}[x]-\frac{1}{2}\sqrt{-1+8\,\text{Cos}[x]^2}\,\text{Sin}[x]$$

Result (type 3, 131 leaves):

$$\frac{1}{8}\left(-6\,\text{ArcTan}\left[\frac{\text{Sin}[x]}{\sqrt{1+2\,\text{Cos}[2x]}}\right]-6\,\text{ArcTan}\left[\frac{\text{Sin}[x]}{\sqrt{3+4\,\text{Cos}[2x]}}\right]-6\,i\,\text{Log}\left[\sqrt{1+2\,\text{Cos}[2x]}+2\,i\,\text{Sin}[x]\right]-\right.$$
$$\left.5\,i\sqrt{2}\,\text{Log}\left[\sqrt{3+4\,\text{Cos}[2x]}+2\,i\sqrt{2}\,\text{Sin}[x]\right]-4\sqrt{1+2\,\text{Cos}[2x]}\,\text{Sin}[x]-4\sqrt{3+4\,\text{Cos}[2x]}\,\text{Sin}[x]\right)$$

Problem 434: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(4-5\,\text{Sec}[x]^2\right)^{3/2}dx$$

Optimal (type 3, 68 leaves, 7 steps):

$$8\,\text{ArcTan}\left[\frac{2\,\text{Tan}[x]}{\sqrt{-1-5\,\text{Tan}[x]^2}}\right]-\frac{7}{2}\sqrt{5}\,\text{ArcTan}\left[\frac{\sqrt{5}\,\text{Tan}[x]}{\sqrt{-1-5\,\text{Tan}[x]^2}}\right]-\frac{5}{2}\text{Tan}[x]\sqrt{-1-5\,\text{Tan}[x]^2}$$

Result (type 3, 115 leaves):

$$-\frac{1}{2\left(-3+2\,\text{Cos}[2x]\right)^{3/2}}\left(-5+4\,\text{Cos}[x]^2\right)\text{Sec}[x]\sqrt{4-5\,\text{Sec}[x]^2}$$
$$\left(7\sqrt{5}\,\text{ArcTan}\left[\frac{\sqrt{5}\,\text{Sin}[x]}{\sqrt{-3+2\,\text{Cos}[2x]}}\right]\text{Cos}[x]^2+16\,i\,\text{Cos}[x]^2\,\text{Log}\left[\sqrt{-3+2\,\text{Cos}[2x]}+2\,i\,\text{Sin}[x]\right]+5\sqrt{-3+2\,\text{Cos}[2x]}\,\text{Sin}[x]\right)$$

Problem 438: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(3+\text{Sin}[x]^2\right)\text{Tan}[x]^3}{\left(-2+\text{Cos}[x]^2\right)\left(5-4\,\text{Sec}[x]^2\right)^{3/2}}dx$$

Optimal (type 3, 73 leaves, 16 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{5-4\,\text{Sec}[x]^2}}{\sqrt{3}}\right]}{6\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{5-4\,\text{Sec}[x]^2}}{\sqrt{5}}\right]}{5\sqrt{5}}-\frac{2}{15\sqrt{5-4\,\text{Sec}[x]^2}}$$

Result (type 3, 234 leaves):

$$\frac{1}{60\left(5-4\,\text{Sec}[x]^2\right)^{3/2}}\text{Sec}[x]^2\left(12-20\,\text{Cos}[2x]+\left(\sqrt{2}\left(-3+5\,\text{Cos}[2x]\right)^{3/2}\left(15\sqrt{3}\,\text{ArcTanh}\left[\frac{\sqrt{-3+5\,\text{Cos}[2x]}}{\sqrt{6}\sqrt{\text{Cos}[x]^2}}\right]\text{Sin}[x]^2-18\sqrt{5}\left(\text{Log}\left[10\,\text{Sin}[x]^2\right]-\text{Log}\left[5\left(-\sqrt{-3+5\,\text{Cos}[2x]}+\text{Cos}[2x]\sqrt{-3+5\,\text{Cos}[2x]}+\sqrt{10}\sqrt{\text{Sin}[x]^2}\sqrt{\text{Sin}[2x]^2}\right)\right]\right)\text{Sin}[x]^2-20\sqrt{3}\,\text{ArcTanh}\left[\frac{\sqrt{6}\,\text{Cos}[x]}{\sqrt{-3+5\,\text{Cos}[2x]}}\right]\text{Sec}[x]\sqrt{\text{Sin}[x]^2}\sqrt{\text{Sin}[2x]^2}\right)\right)\Big/\left(15\sqrt{\text{Sin}[x]^2}\sqrt{\text{Sin}[2x]^2}\right)\right)$$

Problem 439: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Csc}[x]^2\left(\text{Sec}[x]^2-3\,\text{Tan}[x]\sqrt{4\,\text{Sec}[x]^2+5\,\text{Tan}[x]^2}\right)}{\left(4\,\text{Sec}[x]^2+5\,\text{Tan}[x]^2\right)^{3/2}}\,dx$$

Optimal (type 3, 57 leaves, 10 steps):

$$-\frac{3}{4}\text{Log}[\text{Tan}[x]]+\frac{3}{8}\text{Log}\left[4+9\,\text{Tan}[x]^2\right]-\frac{\text{Cot}[x]}{4\sqrt{4+9\,\text{Tan}[x]^2}}-\frac{7\,\text{Tan}[x]}{8\sqrt{4+9\,\text{Tan}[x]^2}}$$

Result (type 3, 116 leaves):

$$\frac{1}{16\sqrt{\frac{13-5\,\text{Cos}[2x]}{1+\text{Cos}[2x]}}}\left(5\,\text{Cot}[x]+6\sqrt{\frac{13-5\,\text{Cos}[2x]}{1+\text{Cos}[2x]}}\,\text{Log}\left[1+7\,\text{Tan}\left[\frac{x}{2}\right]^2+\text{Tan}\left[\frac{x}{2}\right]^4\right]-9\,\text{Csc}[x]\,\text{Sec}[x]-5\,\text{Tan}[x]-6\sqrt{2}\,\text{Log}\left[\text{Tan}\left[\frac{x}{2}\right]\right]\sqrt{-5+13\,\text{Sec}[x]^2+5\,\text{Tan}[x]^2}\right)$$

Problem 442: Result unnecessarily involves higher level functions.

$$\int\frac{\text{Tan}[x]}{\left(a^3+b^3\,\text{Tan}[x]^2\right)^{1/3}}\,dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\left(a^3+b^3\,\text{Tan}[x]^2\right)^{1/3}}{\left(a^3-b^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(a^3-b^3\right)^{1/3}}+\frac{\text{Log}[\text{Cos}[x]]}{2\left(a^3-b^3\right)^{1/3}}+\frac{3\,\text{Log}\left[\left(a^3-b^3\right)^{1/3}-\left(a^3+b^3\,\text{Tan}[x]^2\right)^{1/3}\right]}{4\left(a^3-b^3\right)^{1/3}}$$

Result (type 5, 90 leaves):

$$-\frac{3\left(\frac{a^3+b^3+\left(a^3-b^3\right)\text{Cos}[2x]}{b^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{\left(-a^3+b^3\right)\text{Cos}[x]^2}{b^3}\right]}{2\left(\left(a^3+b^3+\left(a^3-b^3\right)\text{Cos}[2x]\right)\text{Sec}[x]^2\right)^{1/3}}$$

Problem 443: Result unnecessarily involves higher level functions.

$$\int \text{Tan}[x]\left(1-7\,\text{Tan}[x]^2\right)^{2/3}\,dx$$

Optimal (type 3, 69 leaves, 7 steps):

$$2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\left(1-7\,\text{Tan}[x]^2\right)^{1/3}}{\sqrt{3}}\right]+2\,\text{Log}[\text{Cos}[x]]+3\,\text{Log}\left[2-\left(1-7\,\text{Tan}[x]^2\right)^{1/3}\right]+\frac{3}{4}\left(1-7\,\text{Tan}[x]^2\right)^{2/3}$$

Result (type 5, 42 leaves):

$$-\frac{3}{4}\left(-1+\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{1}{8}\left(-3+4\,\text{Cos}[2x]\right)\text{Sec}[x]^2\right]\right)\left(1-7\,\text{Tan}[x]^2\right)^{2/3}$$

Problem 444: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Cot}[x]}{\left(a^4+b^4\,\text{Csc}[x]^2\right)^{1/4}}\,dx$$

Optimal (type 3, 52 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{\left(a^4+b^4\,\text{Csc}[x]^2\right)^{1/4}}{a}\right]}{a}+\frac{\text{ArcTanh}\left[\frac{\left(a^4+b^4\,\text{Csc}[x]^2\right)^{1/4}}{a}\right]}{a}$$

Result (type 5, 84 leaves):

$$-\frac{\left(-a^4-2\,b^4+a^4\,\text{Cos}[2x]\right)\text{Csc}[x]^2\,\text{Hypergeometric2F1}\left[\frac{3}{4},1,\frac{7}{4},-\frac{\left(-a^4-2\,b^4+a^4\,\text{Cos}[2x]\right)\text{Csc}[x]^2}{2\,a^4}\right]}{3\,a^4\left(a^4+b^4\,\text{Csc}[x]^2\right)^{1/4}}$$

Problem 445: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{Cot}[x]}{\left(a^4-b^4 \operatorname{Csc}[x]^2\right)^{1/4}} \, dx$$

Optimal (type 3, 54 leaves, 6 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\left(a^4-b^4 \operatorname{Csc}[x]^2\right)^{1/4}}{a}\right]}{a}+\frac{\operatorname{ArcTanh}\left[\frac{\left(a^4-b^4 \operatorname{Csc}[x]^2\right)^{1/4}}{a}\right]}{a}$$

Result (type 5, 85 leaves):

$$-\frac{\left(-a^4+2 b^4+a^4 \operatorname{Cos}[2 x]\right) \operatorname{Csc}[x]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -\frac{\left(-a^4+2 b^4+a^4 \operatorname{Cos}[2 x]\right) \operatorname{Csc}[x]^2}{2 a^4}\right]}{3 a^4 \left(a^4-b^4 \operatorname{Csc}[x]^2\right)^{1/4}}$$

Problem 446: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[x]^2 \operatorname{Tan}[x] \left(\left(1-3 \operatorname{Sec}[x]^2\right)^{1/3} \operatorname{Sin}[x]^2+3 \operatorname{Tan}[x]^2\right)}{\left(1-3 \operatorname{Sec}[x]^2\right)^{5/6} \left(1-\sqrt{1-3 \operatorname{Sec}[x]^2}\right)} \, dx$$

Optimal (type 3, 133 leaves, 29 steps):

$$\sqrt{3} \operatorname{ArcTan}\left[\frac{1+2 \left(1-3 \operatorname{Sec}[x]^2\right)^{1/6}}{\sqrt{3}}\right]+\frac{1}{4} \operatorname{Log}\left[\operatorname{Sec}[x]^2\right]-\frac{3}{2} \operatorname{Log}\left[1-\left(1-3 \operatorname{Sec}[x]^2\right)^{1/6}\right]+\frac{1}{3} \operatorname{Log}\left[1-\sqrt{1-3 \operatorname{Sec}[x]^2}\right]-\left(1-3 \operatorname{Sec}[x]^2\right)^{1/6}-\frac{1}{4} \left(1-3 \operatorname{Sec}[x]^2\right)^{2/3}+\frac{1}{2 \left(1-\sqrt{1-3 \operatorname{Sec}[x]^2}\right)}$$

Result (type 6, 4397 leaves):

$$-\left(\left(3 \left(6+\left(\frac{-5+\operatorname{Cos}[2 x]}{1+\operatorname{Cos}[2 x]}\right)^{1/3}+\operatorname{Cos}[2 x] \left(\frac{-5+\operatorname{Cos}[2 x]}{1+\operatorname{Cos}[2 x]}\right)^{1/3}\right) \left(3 \operatorname{Sec}[x]^2+\left(1-3 \operatorname{Sec}[x]^2\right)^{1/3}\right)\right.\right.$$
$$\operatorname{Sin}[x]^2 \operatorname{Tan}[x] \left(-2-3 \operatorname{Tan}[x]^2\right)^{5/6} \left(1+\operatorname{Tan}[x]^2\right) \left(2+3 \operatorname{Tan}[x]^2\right) \left(-8 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2} \operatorname{Tan}[x]^2, -\operatorname{Tan}[x]^2\right]+\right.$$
$$\left.4 \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2} \operatorname{Tan}[x]^2, -\operatorname{Tan}[x]^2\right] \operatorname{Tan}[x]^2+3 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2} \operatorname{Tan}[x]^2, -\operatorname{Tan}[x]^2\right] \operatorname{Tan}[x]^2\right)^2$$

$$\Big(\Big(4\,\text{AppellF1}\big[2,\tfrac{1}{2},2,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]+3\,\text{AppellF1}\big[2,\tfrac{3}{2},1,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\Big)\,\text{Tan}[x]^2$$

$$\Big(30\times 3^{2/3}\,\text{Hypergeometric2F1}\Big[\tfrac{1}{3},\tfrac{1}{3},\tfrac{4}{3},\frac{1}{3+3\,\text{Tan}[x]^2}\Big]\sqrt{-2-3\,\text{Tan}[x]^2}\,\big(1+\text{Tan}[x]^2\big)\left(\frac{2+3\,\text{Tan}[x]^2}{1+\text{Tan}[x]^2}\right)^{1/3}+$$

$$12\times 3^{1/6}\,\text{Hypergeometric2F1}\Big[\tfrac{5}{6},\tfrac{5}{6},\tfrac{11}{6},\frac{1}{3+3\,\text{Tan}[x]^2}\Big]\big(1+\text{Tan}[x]^2\big)\left(\frac{2+3\,\text{Tan}[x]^2}{1+\text{Tan}[x]^2}\right)^{5/6}+$$

$$5\,\Big(2\,\text{Log}\big[1+\text{Tan}[x]^2\big]\big(-2-3\,\text{Tan}[x]^2\big)^{5/6}\big(1+\text{Tan}[x]^2\big)+9\,\text{Tan}[x]^4\Big(4+\sqrt{-2-3\,\text{Tan}[x]^2}\Big)+3\,\text{Tan}[x]^2$$

$$\Big(20-2\big(-2-3\,\text{Tan}[x]^2\big)^{1/3}+5\sqrt{-2-3\,\text{Tan}[x]^2}\Big)+2\Big(12-2\big(-2-3\,\text{Tan}[x]^2\big)^{1/3}+3\sqrt{-2-3\,\text{Tan}[x]^2}+\big(-2-3\,\text{Tan}[x]^2\big)^{5/6}\Big)\Big)\Big)-$$

$$8\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\Big(30\times 3^{2/3}\,\text{Hypergeometric2F1}\Big[\tfrac{1}{3},\tfrac{1}{3},\tfrac{4}{3},\frac{1}{3+3\,\text{Tan}[x]^2}\Big]\sqrt{-2-3\,\text{Tan}[x]^2}$$

$$\big(1+\text{Tan}[x]^2\big)\left(\frac{2+3\,\text{Tan}[x]^2}{1+\text{Tan}[x]^2}\right)^{1/3}+12\times 3^{1/6}\,\text{Hypergeometric2F1}\Big[\tfrac{5}{6},\tfrac{5}{6},\tfrac{11}{6},\frac{1}{3+3\,\text{Tan}[x]^2}\Big]\big(1+\text{Tan}[x]^2\big)\left(\frac{2+3\,\text{Tan}[x]^2}{1+\text{Tan}[x]^2}\right)^{5/6}+$$

$$5\,\Big(2\,\text{Log}\big[1+\text{Tan}[x]^2\big]\big(-2-3\,\text{Tan}[x]^2\big)^{5/6}\big(1+\text{Tan}[x]^2\big)+9\,\text{Tan}[x]^4\Big(4+\sqrt{-2-3\,\text{Tan}[x]^2}\Big)+\text{Tan}[x]^2$$

$$\Big(60-7\big(-2-3\,\text{Tan}[x]^2\big)^{1/3}+15\sqrt{-2-3\,\text{Tan}[x]^2}\Big)+2\Big(12-2\big(-2-3\,\text{Tan}[x]^2\big)^{1/3}+3\sqrt{-2-3\,\text{Tan}[x]^2}+\big(-2-3\,\text{Tan}[x]^2\big)^{5/6}\Big)\Big)\Big)\Big)\Big)\Big/$$

$$\Bigg(10\times 2^{1/6}\left(-1+\sqrt{\frac{-5+\text{Cos}[2\,x]}{1+\text{Cos}[2\,x]}}\right)\big(1-3\,\text{Sec}[x]^2\big)^{5/6}\Big(6+\big(1-3\,\text{Sec}[x]^2\big)^{1/3}+\text{Cos}[2\,x]\,\big(1-3\,\text{Sec}[x]^2\big)^{1/3}\Big)$$

$$\big(-4-6\,\text{Tan}[x]^2\big)^{5/6}$$

$$\Big(-8\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]+$$

$$\Big(4\,\text{AppellF1}\big[2,\tfrac{1}{2},2,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]+3\,\text{AppellF1}\big[2,\tfrac{3}{2},1,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\Big)\,\text{Tan}[x]^2\Big)$$

$$\Big(1152\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]^2\,\text{Tan}[x]^3+2880\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]^2\,\text{Tan}[x]^5-$$

$$1152\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\,\text{AppellF1}\big[2,\tfrac{1}{2},2,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\,\text{Tan}[x]^5-$$

$$864\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\,\text{AppellF1}\big[2,\tfrac{3}{2},1,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\,\text{Tan}[x]^5+$$

$$1728\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]^2\,\text{Tan}[x]^7-2880\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]$$

$$\text{AppellF1}\big[2,\tfrac{1}{2},2,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\,\text{Tan}[x]^7+288\,\text{AppellF1}\big[2,\tfrac{1}{2},2,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]^2\,\text{Tan}[x]^7-$$

$$2160\,\text{AppellF1}\big[1,\tfrac{1}{2},1,2,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\,\text{AppellF1}\big[2,\tfrac{3}{2},1,3,-\tfrac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\big]\,\text{Tan}[x]^7+$$

$$432\,\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^7 +$$

$$162\,\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]^2\text{Tan}[x]^7 - 1728\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]$$

$$\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^9 + 720\,\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]^2\text{Tan}[x]^9 -$$

$$1296\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^9 +$$

$$1080\,\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^9 +$$

$$405\,\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]^2\text{Tan}[x]^9 + 432\,\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]^2\text{Tan}[x]^{11} +$$

$$648\,\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^{11} +$$

$$243\,\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]^2\text{Tan}[x]^{11} +$$

$$720\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]^2\text{Tan}[x]^3\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} -$$

$$192\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^3\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} -$$

$$144\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^3\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} +$$

$$1008\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]^2\text{Tan}[x]^5\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} -$$

$$1032\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^5\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} -$$

$$774\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^5\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} +$$

$$128\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[3, \frac{1}{2}, 3, 4, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^5\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} +$$

$$96\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[3, \frac{3}{2}, 2, 4, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^5\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} +$$

$$108\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[3, \frac{5}{2}, 1, 4, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^5\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} -$$

$$1080\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} +$$

$$96\,\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]^2\text{Tan}[x]^7\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} -$$

$$810\,\text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{3}{2}\text{Tan}[x]^2, -\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2 - 3\,\text{Tan}[x]^2\right)^{1/3} +$$

$144\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$54\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$320\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[3,\frac{1}{2},3,4,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$240\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[3,\frac{3}{2},2,4,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$270\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[3,\frac{5}{2},1,4,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$144\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$216\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$81\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$192\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[3,\frac{1}{2},3,4,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$144\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[3,\frac{3}{2},2,4,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$162\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[3,\frac{5}{2},1,4,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{1/3}+$

$1152\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^3\sqrt{-2-3\,\text{Tan}[x]^2}+$

$2880\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^5\sqrt{-2-3\,\text{Tan}[x]^2}-$

$1152\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^5\sqrt{-2-3\,\text{Tan}[x]^2}-$

$864\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^5\sqrt{-2-3\,\text{Tan}[x]^2}+$

$1728\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^7\sqrt{-2-3\,\text{Tan}[x]^2}-$

$2880\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\sqrt{-2-3\,\text{Tan}[x]^2}+$

$288\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^7\sqrt{-2-3\,\text{Tan}[x]^2}-$

$2160\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\sqrt{-2-3\,\text{Tan}[x]^2}+$

$432\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\sqrt{-2-3\,\text{Tan}[x]^2}+$

$162\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^7\sqrt{-2-3\,\text{Tan}[x]^2}\;-$

$1728\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^9\sqrt{-2-3\,\text{Tan}[x]^2}\;+$

$720\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^9\sqrt{-2-3\,\text{Tan}[x]^2}\;-$

$1296\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^9\sqrt{-2-3\,\text{Tan}[x]^2}\;+$

$1080\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^9\sqrt{-2-3\,\text{Tan}[x]^2}\;+$

$405\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^9\sqrt{-2-3\,\text{Tan}[x]^2}\;+$

$432\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^{11}\sqrt{-2-3\,\text{Tan}[x]^2}\;+$

$648\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^{11}\sqrt{-2-3\,\text{Tan}[x]^2}\;+$

$243\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^{11}\sqrt{-2-3\,\text{Tan}[x]^2}+384\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2$

$\text{Tan}[x]^3\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}+576\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^5\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;-$

$384\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^5\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;-$

$288\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^5\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;-$

$576\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;+$

$96\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;-$

$432\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;+$

$144\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;+$

$54\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^7\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;+$

$144\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;+$

$216\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\;+$

$$81\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{3}{2}\text{Tan}[x]^2,-\text{Tan}[x]^2\right]^2\text{Tan}[x]^9\left(-2-3\,\text{Tan}[x]^2\right)^{5/6}\Bigg)\Bigg)\Bigg)$$

Problem 447: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\text{Sec}[x]^2\left(-\text{Cos}[2x]+2\,\text{Tan}[x]^2\right)}{\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}\,dx$$

Optimal (type 3, 100 leaves, ? steps):

$$2\,\text{ArcTanh}\left[\frac{\text{Tan}[x]}{\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}\right]-\frac{11\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Tan}[x]}{\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}\right]}{4\sqrt{2}}+\frac{\text{Tan}[x]}{2\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}+\frac{2\,\text{Tan}[x]^3}{3\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}+\frac{3\,\text{Tan}[x]}{4\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}$$

Result (type 6, 207 leaves):

$$\Bigg(\left(-\text{Cos}[2x]+2\,\text{Tan}[x]^2\right)\Bigg(-3\,\text{Cot}[x]-4\,\text{Cos}[x]\,\text{Sin}[x]+18\,\text{Sin}[x]^2\,\text{Tan}[x]-4\,\text{Tan}[x]^3-9\,\text{ArcTan}\left[\sqrt{-1+\text{Tan}[x]^2}\right]\text{Cos}[x]\,\text{Sin}[x]\sqrt{-1+\text{Tan}[x]^2}-$$
$$\left(72\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\text{Cot}[x]^2,-\text{Cot}[x]^2\right]\text{Cos}[2x]\,\text{Sin}[x]^2\,\text{Tan}[x]\right)\Big/\Bigg(2\,\text{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},\text{Cot}[x]^2,-\text{Cot}[x]^2\right]+$$
$$\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},\text{Cot}[x]^2,-\text{Cot}[x]^2\right]-3\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\text{Cot}[x]^2,-\text{Cot}[x]^2\right]\text{Tan}[x]^2\Bigg)\Bigg)$$
$$\text{Tan}[2x]^2\Bigg)\Big/\left(6\left(-3+6\,\text{Cos}[2x]+\text{Cos}[4x]\right)\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}\right)$$

Problem 448: Result unnecessarily involves higher level functions.

$$\int\frac{\text{Tan}[x]}{\left(a^3-b^3\,\text{Cos}[x]^n\right)^{4/3}}\,dx$$

Optimal (type 3, 112 leaves, 7 steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{a+2\left(a^3-b^3\,\text{Cos}[x]^n\right)^{1/3}}{\sqrt{3}\,a}\right]}{a^4\,n}-\frac{3}{a^3\,n\left(a^3-b^3\,\text{Cos}[x]^n\right)^{1/3}}+\frac{\text{Log}[\text{Cos}[x]]}{2\,a^4}-\frac{3\,\text{Log}\left[a-\left(a^3-b^3\,\text{Cos}[x]^n\right)^{1/3}\right]}{2\,a^4\,n}$$

Result (type 5, 71 leaves):

$$\frac{3\left(-1+\left(1-\frac{a^3\,\text{Cos}[x]^{-n}}{b^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{a^3\,\text{Cos}[x]^{-n}}{b^3}\right]\right)}{a^3\,n\left(a^3-b^3\,\text{Cos}[x]^n\right)^{1/3}}$$

Problem 449: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(1+2\,\text{Cos}[x]^9\right)^{5/6}\,\text{Tan}[x]\,dx$$

Optimal (type 3, 95 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\left(1+2\,\text{Cos}[x]^9\right)^{1/3}}{\sqrt{3}\,\left(1+2\,\text{Cos}[x]^9\right)^{1/6}}\right]}{3\sqrt{3}}+\frac{1}{3}\,\text{ArcTanh}\left[\left(1+2\,\text{Cos}[x]^9\right)^{1/6}\right]-\frac{1}{9}\,\text{ArcTanh}\left[\sqrt{1+2\,\text{Cos}[x]^9}\,\right]-\frac{2}{15}\left(1+2\,\text{Cos}[x]^9\right)^{5/6}$$

Result (type 5, 579 leaves):

$$\Bigg(\left(128+126\,\text{Cos}[x]+84\,\text{Cos}[3\,x]+36\,\text{Cos}[5\,x]+9\,\text{Cos}[7\,x]+\text{Cos}[9\,x]\right)^{5/6}$$
$$\left(1+\text{Cot}[x]^2\right)^5\,\text{Sin}[x]^2\,\left(1+5\,\text{Cot}[x]^2+10\,\text{Cot}[x]^4+10\,\text{Cot}[x]^6+5\,\text{Cot}[x]^8+\text{Cot}[x]^{10}+2\,\text{Cot}[x]^{10}\sqrt{1+\text{Tan}[x]^2}\right)$$
$$\left(\frac{1+5\,\text{Cot}[x]^2+10\,\text{Cot}[x]^4+10\,\text{Cot}[x]^6+5\,\text{Cot}[x]^8+\text{Cot}[x]^{10}+2\,\text{Cot}[x]^{10}\sqrt{1+\text{Tan}[x]^2}}{\left(1+\text{Cot}[x]^2\right)^5}\right)^{1/6}$$
$$\Bigg(-2\left(1+5\,\text{Tan}[x]^2+10\,\text{Tan}[x]^4+10\,\text{Tan}[x]^6+5\,\text{Tan}[x]^8+\text{Tan}[x]^{10}+2\sqrt{1+\text{Tan}[x]^2}\right)+$$
$$5\times 2^{5/6}\,\text{Hypergeometric2F1}\left[\frac{1}{6},\frac{1}{6},\frac{7}{6},-\frac{1}{2}\left(1+\text{Tan}[x]^2\right)^{9/2}\right]\left(1+\text{Tan}[x]^2\right)^5$$
$$\left(2+\sqrt{1+\text{Tan}[x]^2}+4\,\text{Tan}[x]^2\sqrt{1+\text{Tan}[x]^2}+6\,\text{Tan}[x]^4\sqrt{1+\text{Tan}[x]^2}+4\,\text{Tan}[x]^6\sqrt{1+\text{Tan}[x]^2}+\text{Tan}[x]^8\sqrt{1+\text{Tan}[x]^2}\right)^{1/6}\Bigg)\Bigg)\Bigg/$$
$$\Bigg(480\times 2^{5/6}\left(1+\text{Tan}[x]^2\right)^{9/2}\left(\frac{1+5\,\text{Tan}[x]^2+10\,\text{Tan}[x]^4+10\,\text{Tan}[x]^6+5\,\text{Tan}[x]^8+\text{Tan}[x]^{10}+2\sqrt{1+\text{Tan}[x]^2}}{\left(1+\text{Tan}[x]^2\right)^5}\right)^{1/6}$$
$$\Big(4\,\text{Cot}[x]^8+20\,\text{Cot}[x]^{10}+40\,\text{Cot}[x]^{12}+40\,\text{Cot}[x]^{14}+20\,\text{Cot}[x]^{16}+4\,\text{Cot}[x]^{18}+\sqrt{1+\text{Tan}[x]^2}+9\,\text{Cot}[x]^2\sqrt{1+\text{Tan}[x]^2}+$$
$$36\,\text{Cot}[x]^4\sqrt{1+\text{Tan}[x]^2}+84\,\text{Cot}[x]^6\sqrt{1+\text{Tan}[x]^2}+126\,\text{Cot}[x]^8\sqrt{1+\text{Tan}[x]^2}+126\,\text{Cot}[x]^{10}\sqrt{1+\text{Tan}[x]^2}+$$
$$84\,\text{Cot}[x]^{12}\sqrt{1+\text{Tan}[x]^2}+36\,\text{Cot}[x]^{14}\sqrt{1+\text{Tan}[x]^2}+9\,\text{Cot}[x]^{16}\sqrt{1+\text{Tan}[x]^2}+5\,\text{Cot}[x]^{18}\sqrt{1+\text{Tan}[x]^2}\Big)\Bigg)$$

Problem 451: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[x]^2\,\text{Tan}[x]\,\left(1+\left(1-8\,\text{Tan}[x]^2\right)^{1/3}\right)}{\left(1-8\,\text{Tan}[x]^2\right)^{2/3}}\,dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-\frac{3}{32}\left(1+\left(1-8\,\text{Tan}[x]^2\right)^{1/3}\right)^2$$

Result (type 3, 42 leaves):

$$-\frac{3\,(-7+9\,\text{Cos}[2\,x])\,\text{Sec}[x]^2\left(2+\left(1-8\,\text{Tan}[x]^2\right)^{1/3}\right)}{64\left(1-8\,\text{Tan}[x]^2\right)^{2/3}}$$

Problem 452: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[x]\,\text{Sec}[x]\,\left(1+\left(1-8\,\text{Tan}[x]^2\right)^{1/3}\right)}{\left(1-8\,\text{Tan}[x]^2\right)^{2/3}}\,dx$$

Optimal (type 3, 27 leaves, 15 steps):

$$-\text{Log}[\text{Tan}[x]]+\frac{3}{2}\,\text{Log}\left[1-\left(1-8\,\text{Tan}[x]^2\right)^{1/3}\right]$$

Result (type 5, 93 leaves):

$$-\frac{3\left(8-\text{Cot}[x]^2\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{\text{Cot}[x]^2}{8}\right]}{16\left(1-8\,\text{Tan}[x]^2\right)^{2/3}}-\frac{3\left(8-\text{Cot}[x]^2\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{\text{Cot}[x]^2}{8}\right]}{4\left(1-8\,\text{Tan}[x]^2\right)^{1/3}}$$

Problem 453: Result unnecessarily involves higher level functions.

$$\int \frac{\left(5\,\text{Cos}[x]^2-\sqrt{-1+5\,\text{Sin}[x]^2}\right)\text{Tan}[x]}{\left(-1+5\,\text{Sin}[x]^2\right)^{1/4}\left(2+\sqrt{-1+5\,\text{Sin}[x]^2}\right)}\,dx$$

Optimal (type 3, 101 leaves, 14 steps):

$$-\frac{3\,\text{ArcTan}\left[\frac{\left(-1+5\,\text{Sin}[x]^2\right)^{1/4}}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{\text{ArcTanh}\left[\frac{\left(-1+5\,\text{Sin}[x]^2\right)^{1/4}}{\sqrt{2}}\right]}{2\sqrt{2}}+2\left(-1+5\,\text{Sin}[x]^2\right)^{1/4}-\frac{\left(-1+5\,\text{Sin}[x]^2\right)^{1/4}}{2\left(2+\sqrt{-1+5\,\text{Sin}[x]^2}\right)}$$

Result (type 5, 158 leaves):

$$-\frac{1}{60\left(3-5\,\text{Cos}[2\,x]\right)^{3/4}}\left(3\times 2^{1/4}\left(-3+5\,\text{Cos}[2\,x]\right)\left(8\sqrt{2}+\sqrt{3-5\,\text{Cos}[2\,x]}+10\sqrt{2}\,\text{Cos}[2\,x]\right)\text{Sec}[x]^2-\right.$$
$$30\times 5^{3/4}\sqrt{3-5\,\text{Cos}[2\,x]}\;\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{4\,\text{Sec}[x]^2}{5}\right]\left(\left(-3+5\,\text{Cos}[2\,x]\right)\text{Sec}[x]^2\right)^{1/4}+$$
$$\left.28\times 5^{1/4}\,\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{4\,\text{Sec}[x]^2}{5}\right]\left(2-8\,\text{Tan}[x]^2\right)^{3/4}\right)$$

Problem 454: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Cos}[x]^3\,\text{Cos}[2\,x]^{2/3}\,\text{Sin}[x]\,dx$$

Optimal (type 3, 25 leaves, 4 steps):

$$-\frac{3}{40}\,\text{Cos}[2\,x]^{5/3}-\frac{3}{64}\,\text{Cos}[2\,x]^{8/3}$$

Result (type 5, 140 leaves):

$$-\frac{3}{40}\,\text{Cos}[2\,x]^{5/3}-$$
$$\left(3\,e^{-6\,i\,x}\left(1+e^{4\,i\,x}\right)^{1/3}\left(\left(1+e^{4\,i\,x}\right)^{2/3}\left(1+e^{8\,i\,x}\right)+2\,e^{4\,i\,x}\,\text{Hypergeometric2F1}\left[-\frac{1}{3},\frac{1}{3},\frac{2}{3},-e^{4\,i\,x}\right]+e^{8\,i\,x}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-e^{4\,i\,x}\right]\right)\right)\Big/$$
$$\left(256\times 2^{2/3}\left(e^{-2\,i\,x}+e^{2\,i\,x}\right)^{1/3}\right)$$

Problem 455: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Sin}[x]^6\,\text{Tan}[x]}{\text{Cos}[2\,x]^{3/4}}\,dx$$

Optimal (type 3, 102 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{1-\sqrt{\text{Cos}[2\,x]}}{\sqrt{2}\,\text{Cos}[2\,x]^{1/4}}\right]}{\sqrt{2}}-\frac{\text{ArcTanh}\left[\frac{1+\sqrt{\text{Cos}[2\,x]}}{\sqrt{2}\,\text{Cos}[2\,x]^{1/4}}\right]}{\sqrt{2}}+\frac{7}{4}\,\text{Cos}[2\,x]^{1/4}-\frac{1}{5}\,\text{Cos}[2\,x]^{5/4}+\frac{1}{36}\,\text{Cos}[2\,x]^{9/4}$$

Result (type 5, 59 leaves) :

$$\frac{1}{360} \text{Cos}[2x]^{1/4} (635 - 72 \text{Cos}[2x] + 5 \text{Cos}[4x]) + \frac{2 \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{\text{Sec}[x]^2}{2}\right]}{3 (1 + \text{Cos}[2x])^{3/4}}$$

Problem 456: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\text{Tan}[x] \text{Tan}[2x]} \, dx$$

Optimal (type 3, 17 leaves, 3 steps) :

$$-\text{ArcTanh}\left[\frac{\text{Tan}[x]}{\sqrt{\text{Tan}[x] \text{Tan}[2x]}}\right]$$

Result (type 3, 45 leaves) :

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \text{Cos}[x]}{\sqrt{\text{Cos}[2x]}}\right] \sqrt{\text{Cos}[2x]} \text{Csc}[x] \sqrt{\text{Tan}[x] \text{Tan}[2x]}}{\sqrt{2}}$$

Problem 488: Result more than twice size of optimal antiderivative.

$$\int x \text{Sec}[x] \text{Tan}[x]^3 \, dx$$

Optimal (type 3, 30 leaves, 5 steps) :

$$\frac{5}{6} \text{ArcTanh}[\text{Sin}[x]] - x \text{Sec}[x] + \frac{1}{3} x \text{Sec}[x]^3 - \frac{1}{6} \text{Sec}[x] \text{Tan}[x]$$

Result (type 3, 104 leaves) :

$$-\frac{1}{24} \text{Sec}[x]^3 \left(4x + 12x \text{Cos}[2x] + 5 \text{Cos}[3x] \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + 15 \text{Cos}[x] \left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]\right) - 5 \text{Cos}[3x] \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + 2 \text{Sin}[2x]\right)$$

Problem 506: Unable to integrate problem.

$$\int \left(a^{k x} + a^{l x}\right)^n \, dx$$

Optimal (type 5, 72 leaves, 2 steps) :

$$\frac{\left(1+a^{(k-1)\,x}\right)\,\left(a^{k\,x}+a^{l\,x}\right)^n \text{Hypergeometric2F1}\left[1,\,1+\frac{k\,n}{k-1},\,1+\frac{l\,n}{k-1},\,-a^{(k-1)\,x}\right]}{l\,n\,\text{Log}[a]}$$

Result (type 8, 15 leaves):

$$\int \left(a^{k\,x}+a^{l\,x}\right)^n \, dx$$

Problem 511: Unable to integrate problem.

$$\int \left(a^{k\,x}-a^{l\,x}\right)^n \, dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{\left(1-a^{(k-1)\,x}\right)\,\left(a^{k\,x}-a^{l\,x}\right)^n \text{Hypergeometric2F1}\left[1,\,1+\frac{k\,n}{k-1},\,1+\frac{l\,n}{k-1},\,a^{(k-1)\,x}\right]}{l\,n\,\text{Log}[a]}$$

Result (type 8, 17 leaves):

$$\int \left(a^{k\,x}-a^{l\,x}\right)^n \, dx$$

Problem 523: Result is not expressed in closed-form.

$$\int \frac{e^x}{b+a\,e^{3\,x}} \, dx$$

Optimal (type 3, 100 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{b^{1/3}-2\,a^{1/3}\,e^x}{\sqrt{3}\,b^{1/3}}\right]}{\sqrt{3}\,a^{1/3}\,b^{2/3}}+\frac{\text{Log}\left[b^{1/3}+a^{1/3}\,e^x\right]}{2\,a^{1/3}\,b^{2/3}}-\frac{\text{Log}\left[b+a\,e^{3\,x}\right]}{6\,a^{1/3}\,b^{2/3}}$$

Result (type 7, 36 leaves):

$$\frac{\text{RootSum}\left[b+a\,\#1^3\,\&,\,\frac{-x+\text{Log}\left[e^x-\#1\right]}{\#1^2}\,\&\right]}{3\,a}$$

Problem 528: Result unnecessarily involves higher level functions.

$$\int \left(1-2\,e^{x/3}\right)^{1/4} \, dx$$

Optimal (type 3, 54 leaves, 6 steps):

$12\left(1-2e^{x/3}\right)^{1/4}-6\,\text{ArcTan}\left[\left(1-2e^{x/3}\right)^{1/4}\right]-6\,\text{ArcTanh}\left[\left(1-2e^{x/3}\right)^{1/4}\right]$

Result (type 5, 70 leaves):

$$-\frac{2\left(-6+12e^{x/3}+2^{1/4}\left(2-e^{-x/3}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{e^{-x/3}}{2}\right]\right)}{\left(1-2e^{x/3}\right)^{3/4}}$$

Problem 540: Unable to integrate problem.

$$\int\frac{e^x\left(1-x-x^2\right)}{\sqrt{1-x^2}}\,dx$$

Optimal (type 3, 15 leaves, 1 step):

$e^x\sqrt{1-x^2}$

Result (type 8, 27 leaves):

$$\int\frac{e^x\left(1-x-x^2\right)}{\sqrt{1-x^2}}\,dx$$

Problem 552: Result more than twice size of optimal antiderivative.

$$\int\frac{e^x}{1+\text{Cos}[x]}\,dx$$

Optimal (type 5, 28 leaves, 2 steps):

$(1-i)\,e^{(1+i)\,x}\,\text{Hypergeometric2F1}\left[1-i,2,2-i,-e^{i\,x}\right]$

Result (type 5, 89 leaves):

$$-\frac{1}{1+\text{Cos}[x]}(1+i)\,e^x\,\text{Cos}\left[\frac{x}{2}\right]\left((1+i)\,\text{Cos}\left[\frac{x}{2}\right]\text{Hypergeometric2F1}\left[-i,1,1-i,-e^{i\,x}\right]-e^{i\,x}\,\text{Cos}\left[\frac{x}{2}\right]\text{Hypergeometric2F1}\left[1,1-i,2-i,-e^{i\,x}\right]-(1-i)\,\text{Sin}\left[\frac{x}{2}\right]\right)$$

Problem 553: Result more than twice size of optimal antiderivative.

$$\int\frac{e^x}{1-\text{Cos}[x]}\,dx$$

Optimal (type 5, 26 leaves, 2 steps):

$(-1+i)\, e^{(1+i)x}\, \text{Hypergeometric2F1}[1-i, 2, 2-i, e^{ix}]$

Result (type 5, 84 leaves):

$$\frac{1}{-1+\text{Cos}[x]}(1+i)\, e^{x}\, \text{Sin}\left[\frac{x}{2}\right]\left((1-i)\,\text{Cos}\left[\frac{x}{2}\right]+(1+i)\,\text{Hypergeometric2F1}[-i, 1, 1-i, e^{ix}]\,\text{Sin}\left[\frac{x}{2}\right]+e^{ix}\,\text{Hypergeometric2F1}[1, 1-i, 2-i, e^{ix}]\,\text{Sin}\left[\frac{x}{2}\right]\right)$$

Problem 554: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{x}}{1+\text{Sin}[x]}\, dx$$

Optimal (type 5, 30 leaves, 2 steps):

$(-1+i)\, e^{(1-i)x}\, \text{Hypergeometric2F1}[1+i, 2, 2+i, -i\, e^{-ix}]$

Result (type 5, 61 leaves):

$$\frac{2\, e^{x}\, \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}-(1-i)\left(1-(1-i)\,\text{Hypergeometric2F1}[-i, 1, 1-i, i\,\text{Cos}[x]-\text{Sin}[x]]\right)(\text{Cosh}[x]+\text{Sinh}[x])$$

Problem 555: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{x}}{1-\text{Sin}[x]}\, dx$$

Optimal (type 5, 30 leaves, 2 steps):

$(1+i)\, e^{(1+i)x}\, \text{Hypergeometric2F1}[1-i, 2, 2-i, -i\, e^{ix}]$

Result (type 5, 61 leaves):

$$\frac{2\, e^{x}\, \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]}+(1+i)\left(1-(1+i)\,\text{Hypergeometric2F1}[-i, 1, 1-i, -i\,\text{Cos}[x]+\text{Sin}[x]]\right)(\text{Cosh}[x]+\text{Sinh}[x])$$

Problem 557: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{x}\,(1+\text{Sin}[x])}{1-\text{Cos}[x]}\, dx$$

Optimal (type 5, 41 leaves, 7 steps):

$(-2+2\,i)\,e^{(1+i)\,x}\,\text{Hypergeometric2F1}\left[1-i,\,2,\,2-i,\,e^{i\,x}\right]+\frac{e^x\,\text{Sin}[x]}{1-\text{Cos}[x]}$

Result (type 5, 100 leaves):

$\left(2\,e^x\,\text{Sin}\left[\frac{x}{2}\right]\left(\text{Cos}\left[\frac{x}{2}\right]+2\,i\,\text{Hypergeometric2F1}\left[-i,\,1,\,1-i,\,e^{i\,x}\right]\,\text{Sin}\left[\frac{x}{2}\right]+(1+i)\,e^{i\,x}\,\text{Hypergeometric2F1}\left[1,\,1-i,\,2-i,\,e^{i\,x}\right]\,\text{Sin}\left[\frac{x}{2}\right]\right)\right.$
$\left.(1+\text{Sin}[x])\right)\Big/\left((-1+\text{Cos}[x])\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2\right)$

Problem 559: Result more than twice size of optimal antiderivative.

$\int\frac{e^x\,(1-\text{Sin}[x])}{1+\text{Cos}[x]}\,dx$

Optimal (type 5, 42 leaves, 7 steps):

$(2-2\,i)\,e^{(1+i)\,x}\,\text{Hypergeometric2F1}\left[1-i,\,2,\,2-i,\,-e^{i\,x}\right]-\frac{e^x\,\text{Sin}[x]}{1+\text{Cos}[x]}$

Result (type 5, 87 leaves):

$-\frac{1}{1+\text{Cos}[x]}$
$2\,e^x\,\text{Cos}\left[\frac{x}{2}\right]\left(2\,i\,\text{Cos}\left[\frac{x}{2}\right]\,\text{Hypergeometric2F1}\left[-i,\,1,\,1-i,\,-e^{i\,x}\right]-(1+i)\,e^{i\,x}\,\text{Cos}\left[\frac{x}{2}\right]\,\text{Hypergeometric2F1}\left[1,\,1-i,\,2-i,\,-e^{i\,x}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)$

Problem 574: Result more than twice size of optimal antiderivative.

$\int\text{Sech}[x]\,dx$

Optimal (type 3, 3 leaves, 1 step):

$\text{ArcTan}[\text{Sinh}[x]]$

Result (type 3, 9 leaves):

$2\,\text{ArcTan}\left[\text{Tanh}\left[\frac{x}{2}\right]\right]$

Problem 575: Result more than twice size of optimal antiderivative.

$\int\text{Csch}[x]\,dx$

Optimal (type 3, 5 leaves, 1 step):

$-\text{ArcTanh}[\text{Cosh}[x]]$

Result (type 3, 17 leaves):

$-\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]$

Problem 579: Result more than twice size of optimal antiderivative.

$\int \text{Csch}[x]^3\, dx$

Optimal (type 3, 16 leaves, 2 steps):

$\frac{1}{2}\text{ArcTanh}[\text{Cosh}[x]]-\frac{1}{2}\text{Coth}[x]\,\text{Csch}[x]$

Result (type 3, 47 leaves):

$-\frac{1}{8}\text{Csch}\left[\frac{x}{2}\right]^2+\frac{1}{2}\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\frac{1}{2}\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]-\frac{1}{8}\text{Sech}\left[\frac{x}{2}\right]^2$

Problem 592: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$\int \frac{\text{Cosh}[x]\,(-\text{Cosh}[2x]+\text{Tanh}[x])}{\sqrt{\text{Sinh}[2x]}\,\left(\text{Sinh}[x]^2+\text{Sinh}[2x]\right)}\, dx$

Optimal (type 3, 69 leaves, 8 steps):

$\sqrt{2}\,\text{ArcTan}\left[\text{Sech}[x]\sqrt{\text{Cosh}[x]\,\text{Sinh}[x]}\right]+\frac{1}{6}\text{ArcTan}\left[\frac{\text{Sinh}[x]}{\sqrt{\text{Sinh}[2x]}}\right]-\frac{1}{3}\sqrt{2}\,\text{ArcTanh}\left[\text{Sech}[x]\sqrt{\text{Cosh}[x]\,\text{Sinh}[x]}\right]+\frac{\text{Cosh}[x]}{\sqrt{\text{Sinh}[2x]}}$

Result (type 4, 487 leaves):

$$-\frac{\mathrm{Coth}[x]\,\sqrt{\mathrm{Sinh}[2\,x]}\,\left(-\mathrm{Cosh}[2\,x]+\mathrm{Tanh}[x]\right)}{\mathrm{Cosh}[x]+\mathrm{Cosh}[3\,x]-2\,\mathrm{Sinh}[x]}+$$

$$\frac{1}{2\,\left(\mathrm{Cosh}[x]+\mathrm{Cosh}[3\,x]-2\,\mathrm{Sinh}[x]\right)}\,\mathrm{Cosh}[x]\,\Bigg(-\Bigg(\Bigg(6\,(-1)^{1/4}\,\sqrt{1+\mathrm{Coth}\left[\frac{x}{2}\right]^2}\,\Bigg(\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}}\right],\,-1\right]-$$

$$\mathrm{EllipticPi}\left[-(-1)^{1/6},\,\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}}\right],\,-1\right]-\mathrm{EllipticPi}\left[-(-1)^{5/6},\,\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}}\right],\,-1\right]\Bigg)$$

$$\sqrt{\mathrm{Sinh}[2\,x]}\,\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}\,\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]+\mathrm{Tanh}\left[\frac{x}{2}\right]^3}\Bigg)\Bigg/\left(\left(1+\mathrm{Cosh}[x]\right)\,\sqrt{\frac{\mathrm{Sinh}[2\,x]}{\left(1+\mathrm{Cosh}[x]\right)^2}}\,\left(1+\mathrm{Tanh}\left[\frac{x}{2}\right]^2\right)\right)\Bigg)+$$

$$\Bigg(16\,(-1)^{5/12}\,\Bigg(\left(3-3\,\mathrm{i}\,\sqrt{3}\right)\,\mathrm{EllipticPi}\left[-\mathrm{i},\,\mathrm{i}\,\mathrm{ArcSinh}\left[(-1)^{1/4}\,\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}\right],\,-1\right]+2\,\left(-1+(-1)^{1/3}\right)$$

$$\mathrm{EllipticPi}\left[\mathrm{i},\,\mathrm{ArcSin}\left[(-1)^{3/4}\,\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}\right],\,-1\right]+\mathrm{i}\,\left(\mathrm{i}+\sqrt{3}\right)\,\mathrm{EllipticPi}\left[-(-1)^{1/6},\,\mathrm{i}\,\mathrm{ArcSinh}\left[(-1)^{1/4}\,\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}\right],\,-1\right]+$$

$$2\,\left(-1+(-1)^{1/3}\right)\,\mathrm{EllipticPi}\left[-(-1)^{5/6},\,\mathrm{i}\,\mathrm{ArcSinh}\left[(-1)^{1/4}\,\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}\right],\,-1\right]\Bigg)\,\mathrm{Sinh}[2\,x]^{3/2}\,\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]+\mathrm{Tanh}\left[\frac{x}{2}\right]^3}\Bigg)\Bigg/$$

$$\left(3\,\left(-\mathrm{i}+\sqrt{3}\right)\,\left(1+\mathrm{Cosh}[x]\right)^3\,\left(\frac{\mathrm{Sinh}[2\,x]}{\left(1+\mathrm{Cosh}[x]\right)^2}\right)^{3/2}\,\sqrt{\mathrm{Tanh}\left[\frac{x}{2}\right]}\,\sqrt{1+\mathrm{Tanh}\left[\frac{x}{2}\right]^2}\right)\Bigg)\,\left(-\mathrm{Cosh}[2\,x]+\mathrm{Tanh}[x]\right)$$

Problem 601: Result more than twice size of optimal antiderivative.

$$\int e^{-2\,x}\,\mathrm{Sech}[x]^4\,dx$$

Optimal (type 3, 13 leaves, 3 steps) :

$$-\frac{8}{3\left(1+e^{2x}\right)^3}$$

Result (type 3, 32 leaves):

$$\frac{8\, e^{2x}\left(3+3\, e^{2x}+e^{4x}\right)}{3\left(1+e^{2x}\right)^3}$$

Problem 622: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x\sqrt{a^2+\text{Log}[x]^2}}\, dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\text{ArcTanh}\left[\frac{\text{Log}[x]}{\sqrt{a^2+\text{Log}[x]^2}}\right]$$

Result (type 3, 46 leaves):

$$-\frac{1}{2}\,\text{Log}\left[1-\frac{\text{Log}[x]}{\sqrt{a^2+\text{Log}[x]^2}}\right]+\frac{1}{2}\,\text{Log}\left[1+\frac{\text{Log}[x]}{\sqrt{a^2+\text{Log}[x]^2}}\right]$$

Problem 623: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x\sqrt{-a^2+\text{Log}[x]^2}}\, dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\text{ArcTanh}\left[\frac{\text{Log}[x]}{\sqrt{-a^2+\text{Log}[x]^2}}\right]$$

Result (type 3, 50 leaves):

$$-\frac{1}{2}\,\text{Log}\left[1-\frac{\text{Log}[x]}{\sqrt{-a^2+\text{Log}[x]^2}}\right]+\frac{1}{2}\,\text{Log}\left[1+\frac{\text{Log}[x]}{\sqrt{-a^2+\text{Log}[x]^2}}\right]$$

Problem 627: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x \operatorname{Log}[x] \sqrt{-a^2+\operatorname{Log}[x]^2}} \, dx$$

Optimal (type 3, 23 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{-a^2+\operatorname{Log}[x]^2}}{a}\right]}{a}$$

Result (type 3, 38 leaves):

$$-\frac{i \operatorname{Log}\left[-\frac{2 i a}{\operatorname{Log}[x]}+\frac{2 \sqrt{-a^2+\operatorname{Log}[x]^2}}{\operatorname{Log}[x]}\right]}{a}$$

Problem 689: Result more than twice size of optimal antiderivative.

$$\int \frac{x^6 \operatorname{ArcSec}[x]}{\left(-1+x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 175 leaves, 16 steps):

$$\frac{\sqrt{x^2}\left(2-3 x^2\right)}{6\left(-1+x^2\right)}-\frac{13}{6} \operatorname{ArcCoth}\left[\sqrt{x^2}\right]-\frac{5 x^3 \operatorname{ArcSec}[x]}{6\left(-1+x^2\right)^{3/2}}+\frac{x^5 \operatorname{ArcSec}[x]}{2\left(-1+x^2\right)^{3/2}}-\frac{5 x \operatorname{ArcSec}[x]}{2 \sqrt{-1+x^2}}-$$
$$\frac{5 i \sqrt{x^2} \operatorname{ArcSec}[x] \operatorname{ArcTan}\left[e^{i \operatorname{ArcSec}[x]}\right]}{x}+\frac{5 i \sqrt{x^2} \operatorname{PolyLog}\left[2,-i e^{i \operatorname{ArcSec}[x]}\right]}{2 x}-\frac{5 i \sqrt{x^2} \operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcSec}[x]}\right]}{2 x}$$

Result (type 4, 383 leaves):

$$-\frac{1}{96\left(-1+x^2\right)^{3/2}}x^5\left(22\,\text{ArcSec}[x]+40\,\text{ArcSec}[x]\,\text{Cos}[2\,\text{ArcSec}[x]]-30\,\text{ArcSec}[x]\,\text{Cos}[4\,\text{ArcSec}[x]]-30\sqrt{1-\frac{1}{x^2}}\,\text{ArcSec}[x]\,\text{Log}\left[1-i\,e^{i\,\text{ArcSec}[x]}\right]+\right.$$
$$30\sqrt{1-\frac{1}{x^2}}\,\text{ArcSec}[x]\,\text{Log}\left[1+i\,e^{i\,\text{ArcSec}[x]}\right]+26\sqrt{1-\frac{1}{x^2}}\,\text{Log}\left[\text{Cos}\left[\frac{\text{ArcSec}[x]}{2}\right]\right]-26\sqrt{1-\frac{1}{x^2}}\,\text{Log}\left[\text{Sin}\left[\frac{\text{ArcSec}[x]}{2}\right]\right]+16\,\text{Sin}[2\,\text{ArcSec}[x]]-$$
$$60\,i\sqrt{1-\frac{1}{x^2}}\,\text{PolyLog}\left[2,-i\,e^{i\,\text{ArcSec}[x]}\right]\text{Sin}[2\,\text{ArcSec}[x]]^2+60\,i\sqrt{1-\frac{1}{x^2}}\,\text{PolyLog}\left[2,i\,e^{i\,\text{ArcSec}[x]}\right]\text{Sin}[2\,\text{ArcSec}[x]]^2-$$
$$15\,\text{ArcSec}[x]\,\text{Log}\left[1-i\,e^{i\,\text{ArcSec}[x]}\right]\text{Sin}[3\,\text{ArcSec}[x]]+15\,\text{ArcSec}[x]\,\text{Log}\left[1+i\,e^{i\,\text{ArcSec}[x]}\right]\text{Sin}[3\,\text{ArcSec}[x]]+$$
$$13\,\text{Log}\left[\text{Cos}\left[\frac{\text{ArcSec}[x]}{2}\right]\right]\text{Sin}[3\,\text{ArcSec}[x]]-13\,\text{Log}\left[\text{Sin}\left[\frac{\text{ArcSec}[x]}{2}\right]\right]\text{Sin}[3\,\text{ArcSec}[x]]-4\,\text{Sin}[4\,\text{ArcSec}[x]]+$$
$$15\,\text{ArcSec}[x]\,\text{Log}\left[1-i\,e^{i\,\text{ArcSec}[x]}\right]\text{Sin}[5\,\text{ArcSec}[x]]-15\,\text{ArcSec}[x]\,\text{Log}\left[1+i\,e^{i\,\text{ArcSec}[x]}\right]\text{Sin}[5\,\text{ArcSec}[x]]-$$
$$\left.13\,\text{Log}\left[\text{Cos}\left[\frac{\text{ArcSec}[x]}{2}\right]\right]\text{Sin}[5\,\text{ArcSec}[x]]+13\,\text{Log}\left[\text{Sin}\left[\frac{\text{ArcSec}[x]}{2}\right]\right]\text{Sin}[5\,\text{ArcSec}[x]]\right)$$

Problem 698: Result more than twice size of optimal antiderivative.

$$\int -\frac{\text{ArcTan}[a-x]}{a+x}\,dx$$

Optimal (type 4, 122 leaves, 5 steps):

$$\text{ArcTan}[a-x]\,\text{Log}\left[\frac{2}{1-i\,(a-x)}\right]-\text{ArcTan}[a-x]\,\text{Log}\left[-\frac{2\,(a+x)}{(i-2\,a)\,(1-i\,(a-x))}\right]-$$
$$\frac{1}{2}\,i\,\text{PolyLog}\left[2,1-\frac{2}{1-i\,(a-x)}\right]+\frac{1}{2}\,i\,\text{PolyLog}\left[2,1+\frac{2\,(a+x)}{(i-2\,a)\,(1-i\,(a-x))}\right]$$

Result (type 4, 256 leaves):

$$-\text{ArcTan}[a-x]\left(\frac{1}{2}\text{Log}\left[1+a^2-2\,a\,x+x^2\right]+\text{Log}[-\text{Sin}[\text{ArcTan}[2\,a]-\text{ArcTan}[a-x]]]\right)+$$

$$\frac{1}{2}\left(\frac{1}{4}\,\mathrm{i}\,(\pi-2\,\text{ArcTan}[a-x])^2+\mathrm{i}\,(\text{ArcTan}[2\,a]-\text{ArcTan}[a-x])^2-\right.$$

$$(\pi-2\,\text{ArcTan}[a-x])\,\text{Log}\left[1+e^{-2\,\mathrm{i}\,\text{ArcTan}[a-x]}\right]-2\,(-\text{ArcTan}[2\,a]+\text{ArcTan}[a-x])\,\text{Log}\left[1-e^{2\,\mathrm{i}\,(-\text{ArcTan}[2\,a]+\text{ArcTan}[a-x])}\right]+$$

$$(\pi-2\,\text{ArcTan}[a-x])\,\text{Log}\left[\frac{2}{\sqrt{1+(a-x)^2}}\right]-2\,(\text{ArcTan}[2\,a]-\text{ArcTan}[a-x])\,\text{Log}[-2\,\text{Sin}[\text{ArcTan}[2\,a]-\text{ArcTan}[a-x]]]+$$

$$\left.\mathrm{i}\,\text{PolyLog}\left[2,-e^{-2\,\mathrm{i}\,\text{ArcTan}[a-x]}\right]+\mathrm{i}\,\text{PolyLog}\left[2,e^{2\,\mathrm{i}\,(-\text{ArcTan}[2\,a]+\text{ArcTan}[a-x])}\right]\right)$$

Problem 703: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{ArcSin}[\text{Sinh}[x]]\,\text{Sech}[x]^4\,dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{2}{3}\text{ArcSin}\left[\frac{\text{Cosh}[x]}{\sqrt{2}}\right]+\frac{1}{6}\text{Sech}[x]\,\sqrt{1-\text{Sinh}[x]^2}+\text{ArcSin}[\text{Sinh}[x]]\,\text{Tanh}[x]-\frac{1}{3}\text{ArcSin}[\text{Sinh}[x]]\,\text{Tanh}[x]^3$$

Result (type 3, 66 leaves):

$$\frac{1}{12}\left(8\,\mathrm{i}\,\text{Log}\left[\mathrm{i}\,\sqrt{2}\,\text{Cosh}[x]+\sqrt{3-\text{Cosh}[2\,x]}\right]+\sqrt{6-2\,\text{Cosh}[2\,x]}\,\text{Sech}[x]+4\,\text{ArcSin}[\text{Sinh}[x]]\,(2+\text{Cosh}[2\,x])\,\text{Sech}[x]^2\,\text{Tanh}[x]\right)$$

Problem 704: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{ArcCot}[\text{Cosh}[x]]\,\text{Coth}[x]\,\text{Csch}[x]^3\,dx$$

Optimal (type 3, 36 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{2}}\right]}{6\,\sqrt{2}}+\frac{\text{Coth}[x]}{6}-\frac{1}{3}\text{ArcCot}[\text{Cosh}[x]]\,\text{Csch}[x]^3$$

Result (type 3, 144 leaves):

$$\frac{1}{48}\text{Csch}[x]^3\left(-16\,\text{ArcCot}[\text{Cosh}[x]]-2\,\text{Cosh}[x]+2\,\text{Cosh}[3\,x]-3\,\mathrm{i}\,\sqrt{2}\,\text{ArcTan}\left[1-\mathrm{i}\,\sqrt{2}\,\text{Tanh}\left[\frac{x}{2}\right]\right]\text{Sinh}[x]+\right.$$

$$\left.3\,\mathrm{i}\,\sqrt{2}\,\text{ArcTan}\left[1+\mathrm{i}\,\sqrt{2}\,\text{Tanh}\left[\frac{x}{2}\right]\right]\text{Sinh}[x]+\mathrm{i}\,\sqrt{2}\,\text{ArcTan}\left[1-\mathrm{i}\,\sqrt{2}\,\text{Tanh}\left[\frac{x}{2}\right]\right]\text{Sinh}[3\,x]-\mathrm{i}\,\sqrt{2}\,\text{ArcTan}\left[1+\mathrm{i}\,\sqrt{2}\,\text{Tanh}\left[\frac{x}{2}\right]\right]\text{Sinh}[3\,x]\right)$$

Problem 705: Result more than twice size of optimal antiderivative.

$$\int e^x \text{ArcSin}[\text{Tanh}[x]] \, dx$$

Optimal (type 3, 28 leaves, 5 steps):

$$e^x \text{ArcSin}[\text{Tanh}[x]] - \text{Cosh}[x] \text{Log}\left[1 + e^{2x}\right] \sqrt{\text{Sech}[x]^2}$$

Result (type 3, 64 leaves):

$$e^x \text{ArcSin}\left[\frac{-1 + e^{2x}}{1 + e^{2x}}\right] - e^{-x} \sqrt{\frac{e^{2x}}{\left(1 + e^{2x}\right)^2}} \left(1 + e^{2x}\right) \text{Log}\left[1 + e^{2x}\right]$$

Test results for the 116 problems in "Welz Problems.m"

Problem 2: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-2 \text{Log}\left[-\sqrt{-1 + a x}\right] + \text{Log}[-1 + a x]}{2 \pi \sqrt{-1 + a x}} \, dx$$

Optimal (type 2, 15 leaves, 5 steps):

$$-\frac{2 \sqrt{1 - a x}}{a}$$

Result (type 3, 37 leaves):

$$\frac{\sqrt{-1 + a x} \left(-2 \text{Log}\left[-\sqrt{-1 + a x}\right] + \text{Log}[-1 + a x]\right)}{a \pi}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-1 + x^2}}{(-i + x)^2} \, dx$$

Optimal (type 3, 64 leaves, 6 steps):

$$\frac{\sqrt{-1 + x^2}}{i - x} - \frac{i \text{ArcTan}\left[\frac{1 - i x}{\sqrt{2} \sqrt{-1 + x^2}}\right]}{\sqrt{2}} + \text{ArcTanh}\left[\frac{x}{\sqrt{-1 + x^2}}\right]$$

Result (type 3, 165 leaves):

$$\frac{1}{4}\left(-\frac{4\sqrt{-1+x^2}}{-\mathrm{i}+x}-2\,\mathrm{i}\,\sqrt{2}\,\text{ArcTan}\left[\frac{1}{2}\left(-\mathrm{i}+x-\sqrt{2}\,\sqrt{-1+x^2}\right)\right]+4\,\text{ArcTanh}\left[\frac{2\,x}{\mathrm{i}-x+\sqrt{-1+x^2}}\right]-\sqrt{2}\,\text{Log}[-\mathrm{i}+x]+\sqrt{2}\,\text{Log}\left[-\mathrm{i}-3\,x+2\sqrt{2}\,\sqrt{-1+x^2}\right]+2\,\text{Log}\left[1+2\,\mathrm{i}\,x-2\,x^2+2\,\mathrm{i}\,\sqrt{-1+x^2}-2\,x\,\sqrt{-1+x^2}\right]\right)$$

Problem 9: Unable to integrate problem.

$$\int \frac{1}{\sqrt{-1+x^2}\,\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4\,x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 8, 29 leaves):

$$\int \frac{1}{\sqrt{-1+x^2}\,\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$

Problem 10: Unable to integrate problem.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4\,x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 8, 41 leaves):

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \left(\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{i+x^2}}\right)dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}\,(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}\,(1+x)}+\frac{\text{ArcTanh}\left[\frac{i+x}{\sqrt{1-i}\,\sqrt{-i+x^2}}\right]}{(1-i)^{3/2}\sqrt{2}}-\frac{\text{ArcTanh}\left[\frac{i-x}{\sqrt{1+i}\,\sqrt{i+x^2}}\right]}{(1+i)^{3/2}\sqrt{2}}$$

Result (type 3, 403 leaves):

$$-\frac{1}{4\sqrt{2}\,(1+x)}\left((2+2i)\sqrt{-i+x^2}+(2-2i)\sqrt{i+x^2}+2\sqrt{1-i}\,(1+x)\,\text{ArcTan}\left[\frac{1+x^2+2i\sqrt{1-i}\,\sqrt{-i+x^2}}{(1-2i)-2ix+x^2}\right]+\right.$$
$$2\sqrt{1+i}\,(1+x)\,\text{ArcTan}\left[\frac{1+x^2-2i\sqrt{1+i}\,\sqrt{i+x^2}}{(1+2i)+2ix+x^2}\right]-i\sqrt{1-i}\,\text{Log}\left[(1+x)^2\right]+i\sqrt{1+i}\,\text{Log}\left[(1+x)^2\right]-i\sqrt{1-i}\,x\,\text{Log}\left[(1+x)^2\right]+$$
$$i\sqrt{1+i}\,x\,\text{Log}\left[(1+x)^2\right]+i\sqrt{1-i}\,\text{Log}\left[i-(2-i)x^2+2\sqrt{1-i}\,x\sqrt{-i+x^2}\right]+i\sqrt{1-i}\,x\,\text{Log}\left[i-(2-i)x^2+2\sqrt{1-i}\,x\sqrt{-i+x^2}\right]-$$
$$\left.i\sqrt{1+i}\,\text{Log}\left[-i-(2+i)x^2+2\sqrt{1+i}\,x\sqrt{i+x^2}\right]-i\sqrt{1+i}\,x\,\text{Log}\left[-i-(2+i)x^2+2\sqrt{1+i}\,x\sqrt{i+x^2}\right]\right)$$

Problem 12: Unable to integrate problem.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Optimal (type 3, 125 leaves, 7 steps):

$$-\frac{\sqrt{1-i x^2}}{2\,(1+x)}-\frac{\sqrt{1+i x^2}}{2\,(1+x)}-\frac{1}{4}(1-i)^{3/2}\,\text{ArcTanh}\left[\frac{1+i x}{\sqrt{1-i}\,\sqrt{1-i x^2}}\right]-\frac{1}{4}(1+i)^{3/2}\,\text{ArcTanh}\left[\frac{1-i x}{\sqrt{1+i}\,\sqrt{1+i x^2}}\right]$$

Result (type 8, 34 leaves):

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2 \sqrt{1+x^4}} \, dx$$

Problem 13: Unable to integrate problem.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x) \sqrt{1+x^4}} \, dx$$

Optimal (type 3, 81 leaves, 5 steps):

$$-\frac{1}{2} \sqrt{1-i}\, \text{ArcTanh}\left[\frac{1+i x}{\sqrt{1-i} \sqrt{1-i x^2}}\right]-\frac{1}{2} \sqrt{1+i}\, \text{ArcTanh}\left[\frac{1-i x}{\sqrt{1+i} \sqrt{1+i x^2}}\right]$$

Result (type 8, 34 leaves):

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x) \sqrt{1+x^4}} \, dx$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx$$

Optimal (type 3, 31 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\, x}{\sqrt{x^2+\sqrt{1+x^4}}}\right]}{\sqrt{2}}$$

Result (type 3, 145 leaves):

$$-\frac{x \left(1+x^4+x^2 \sqrt{1+x^4}\right) \left(\text{Log}\left[1-\frac{\sqrt{x^2 \left(x^2+\sqrt{1+x^4}\right)}}{\sqrt{2}\, x^2}\right]-\text{Log}\left[1+\frac{\sqrt{x^2 \left(x^2+\sqrt{1+x^4}\right)}}{\sqrt{2}\, x^2}\right]\right)}{2 \sqrt{2} \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}} \sqrt{x^2 \left(x^2+\sqrt{1+x^4}\right)}}$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right]}{\sqrt{2}}$$

Result (type 3, 162 leaves):

$$\frac{x\left(1+2x^4-2x^2\sqrt{1+x^4}\right)^2\left(1+x^4-x^2\sqrt{1+x^4}\right)\text{ArcSin}\left[x^2-\sqrt{1+x^4}\right]}{\sqrt{2}\sqrt{-x^2+\sqrt{1+x^4}}\sqrt{x^2\left(-x^2+\sqrt{1+x^4}\right)}\left(-4x^2-12x^6-8x^{10}+\sqrt{1+x^4}+8x^4\sqrt{1+x^4}+8x^8\sqrt{1+x^4}\right)}$$

Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx$$

Optimal (type 3, 86 leaves, 6 steps):

$$\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}\,(1+x)}{\sqrt{1-x+x^2}}\right]-\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{1-x+x^2}}\right]}{\sqrt{6}}$$

Result (type 3, 961 leaves):

$$\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\frac{1}{4\sqrt{3-3\,i\,\sqrt{3}}}$$
$$\left(7-i\sqrt{3}\right)\text{ArcTan}\Big[\left(3\left(-17-64\,i\sqrt{3}+\left(94+32\,i\sqrt{3}\right)x+\left(-103-36\,i\sqrt{3}\right)x^2+14\left(7-2\,i\sqrt{3}\right)x^3+\left(-21-4\,i\sqrt{3}\right)x^4\right)\right)\Big/$$
$$\left(96\,i+67\sqrt{3}+\left(84\,i-113\sqrt{3}\right)x^4-52\sqrt{3-3\,i\sqrt{3}}\sqrt{1-x+x^2}+2x\left(132\,i-69\sqrt{3}+26\sqrt{3-3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)+\right.$$
$$\left.x^2\left(-180\,i-59\sqrt{3}+52\sqrt{3-3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)+2x^3\left(138\,i+21\sqrt{3}+52\sqrt{3-3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)\right)\Big]-\frac{1}{4\sqrt{3+3\,i\sqrt{3}}}$$
$$i\left(-7\,i+\sqrt{3}\right)\text{ArcTan}\Big[\left(3\left(-17+64\,i\sqrt{3}+\left(94-32\,i\sqrt{3}\right)x+\left(-103+36\,i\sqrt{3}\right)x^2+14\left(7+2\,i\sqrt{3}\right)x^3+\left(-21+4\,i\sqrt{3}\right)x^4\right)\right)\Big/$$
$$\left(96\,i-67\sqrt{3}+\left(84\,i+113\sqrt{3}\right)x^4+52\sqrt{3+3\,i\sqrt{3}}\sqrt{1-x+x^2}+x^2\left(-180\,i+59\sqrt{3}-52\sqrt{3+3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)+\right.$$
$$\left.x\left(264\,i+138\sqrt{3}-52\sqrt{3+3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)-2x^3\left(-138\,i+21\sqrt{3}+52\sqrt{3+3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)\right)\Big]-$$
$$\frac{\left(-7\,i+\sqrt{3}\right)\text{Log}\left[16\left(1+x+x^2\right)^2\right]}{8\sqrt{3+3\,i\sqrt{3}}}-\frac{\left(7\,i+\sqrt{3}\right)\text{Log}\left[16\left(1+x+x^2\right)^2\right]}{8\sqrt{3-3\,i\sqrt{3}}}+\frac{1}{8\sqrt{3-3\,i\sqrt{3}}}$$
$$\left(7\,i+\sqrt{3}\right)\text{Log}\Big[\left(1+x+x^2\right)\left(11\,i+4\sqrt{3}+\left(11\,i+4\sqrt{3}\right)x^2+10\,i\sqrt{1-i\sqrt{3}}\sqrt{1-x+x^2}-x\left(17\,i+4\sqrt{3}+8\,i\sqrt{1-i\sqrt{3}}\sqrt{1-x+x^2}\right)\right)\Big]+$$
$$\frac{1}{8\sqrt{3+3\,i\sqrt{3}}}$$
$$\left(-7\,i+\sqrt{3}\right)\text{Log}\Big[\left(1+x+x^2\right)\left(-11\,i+4\sqrt{3}+\left(-11\,i+4\sqrt{3}\right)x^2-10\,i\sqrt{1+i\sqrt{3}}\sqrt{1-x+x^2}+x\left(17\,i-4\sqrt{3}+8\,i\sqrt{1+i\sqrt{3}}\sqrt{1-x+x^2}\right)\right)\Big]$$

Problem 33: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x\left(1-x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\Big[\frac{1+2\left(1-x^2\right)^{1/3}}{\sqrt{3}}\Big]-\frac{\text{Log}[x]}{2}+\frac{3}{4}\text{Log}\Big[1-\left(1-x^2\right)^{1/3}\Big]$$

Result (type 5, 41 leaves):

$$-\frac{3\left(\frac{-1+x^2}{x^2}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1}{x^2}\right]}{2\left(1-x^2\right)^{1/3}}$$

Problem 34: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$-\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1+2\left(1-x^2\right)^{1/3}}{\sqrt{3}}\right]-\frac{\text{Log}[x]}{2}+\frac{3}{4}\text{Log}\left[1-\left(1-x^2\right)^{1/3}\right]$$

Result (type 5, 41 leaves):

$$-\frac{3\left(\frac{-1+x^2}{x^2}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{x^2}\right]}{4\left(1-x^2\right)^{2/3}}$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]$$

Result (type 5, 39 leaves):

$$-\frac{\left(\frac{-1+x^3}{x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1}{x^3}\right]}{\left(1-x^3\right)^{1/3}}$$

Problem 37: Unable to integrate problem.

$$\int \frac{1}{(1+x)\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 97 leaves, 1 step):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{4\times 2^{1/3}}+\frac{3\,\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{(1+x)\left(1-x^3\right)^{1/3}}\,dx$$

Problem 38: Unable to integrate problem.

$$\int \frac{x}{(1+x)\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 145 leaves, 3 steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{4\times 2^{1/3}}+\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]-\frac{3\,\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 8, 20 leaves):

$$\int \frac{x}{(1+x)\left(1-x^3\right)^{1/3}}\,dx$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(2-3x+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}(2-x)}{\sqrt{3}\left(2-3x+x^2\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}[2-x]}{4\times 2^{1/3}}-\frac{\text{Log}[x]}{2\times 2^{1/3}}+\frac{3\,\text{Log}\left[2-x-2^{2/3}\left(2-3x+x^2\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 109 leaves):

$$-\left(\left(15\,x\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{1}{x},\frac{2}{x}\right]\right)\Big/\left(2\left(2-3x+x^2\right)^{1/3}\left(5\,x\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{1}{x},\frac{2}{x}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{1}{x},\frac{2}{x}\right]+\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{1}{x},\frac{2}{x}\right]\right)\right)\right)$$

Problem 40: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(-5+7x-3x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 81 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,(-1+x)}{\sqrt{3}\,\left(-5+7x-3x^2+x^3\right)^{1/3}}\right]+\frac{1}{4}\,\text{Log}[1-x]-\frac{3}{4}\,\text{Log}\left[1-x+\left(-5+7x-3x^2+x^3\right)^{1/3}\right]$$

Result (type 6, 85 leaves) :

$$\frac{1}{4\,\left(-5+7x-3x^2+x^3\right)^{1/3}}3\,\left((2-i)+i\,x\right)^{1/3}\,\left(i\,(-1+x)\right)^{1/3}\,\left((-1+2i)+x\right)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{1}{4}\,i\,\left((-1+2i)+x\right),-\frac{1}{2}\,i\,\left((-1+2i)+x\right)\right]$$

Problem 41: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(x\,\left(-q+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2x}{\sqrt{3}\,\left(x\,\left(-q+x^2\right)\right)^{1/3}}\right]+\frac{\text{Log}[x]}{4}-\frac{3}{4}\,\text{Log}\left[-x+\left(x\,\left(-q+x^2\right)\right)^{1/3}\right]$$

Result (type 5, 49 leaves) :

$$\frac{3\,x\,\left(\frac{q-x^2}{q}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{x^2}{q}\right]}{2\,\left(-q\,x+x^3\right)^{1/3}}$$

Problem 42: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left((-1+x)\,\left(q-2x+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 79 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,(-1+x)}{\sqrt{3}\,\left((-1+x)\,\left(q-2x+x^2\right)\right)^{1/3}}\right]+\frac{1}{4}\,\text{Log}[1-x]-\frac{3}{4}\,\text{Log}\left[1-x+\left((-1+x)\,\left(q-2x+x^2\right)\right)^{1/3}\right]$$

Result (type 5, 61 leaves) :

$$\frac{3\,(-1+x)\,\left(\frac{q+(-2+x)\,x}{-1+q}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{(-1+x)^2}{-1+q}\right]}{2\,\left((-1+x)\,\left(q+(-2+x)\,x\right)\right)^{1/3}}$$

Problem 43: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x \left((-1+x) \left(q-2 q x+x^2\right) \right)^{1/3}} \, dx$$

Optimal (type 3, 118 leaves, ? steps):

$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 q^{1/3} (-1+x)}{\sqrt{3} \left((-1+x) \left(q-2 q x+x^2\right)\right)^{1/3}}\right]}{2 q^{1/3}}+\frac{\text{Log}[1-x]}{4 q^{1/3}}+\frac{\text{Log}[x]}{2 q^{1/3}}-\frac{3 \text{Log}\left[-q^{1/3} (-1+x)+\left((-1+x) \left(q-2 q x+x^2\right)\right)^{1/3}\right]}{4 q^{1/3}}$$

Result (type 5, 72 leaves) :

$$\frac{3 (-1+x) \left(-\frac{q-2 q x+x^2}{(-1+q) x^2}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{q (-1+x)^2}{(-1+q) x^2}\right]}{2 \left((-1+x) \left(q-2 q x+x^2\right)\right)^{1/3}}$$

Problem 44: Unable to integrate problem.

$$\int \frac{2-(1+k) x}{\left((1-x) x (1-k x)\right)^{1/3} (1-(1+k) x)} \, dx$$

Optimal (type 3, 111 leaves, ? steps):

$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1+\frac{2 k^{1/3} x}{((1-x) x (1-k x))^{1/3}}}{\sqrt{3}}\right]}{k^{1/3}}+\frac{\text{Log}[x]}{2 k^{1/3}}+\frac{\text{Log}[1-(1+k) x]}{2 k^{1/3}}-\frac{3 \text{Log}\left[-k^{1/3} x+((1-x) x (1-k x))^{1/3}\right]}{2 k^{1/3}}$$

Result (type 8, 38 leaves) :

$$\int \frac{2-(1+k) x}{\left((1-x) x (1-k x)\right)^{1/3} (1-(1+k) x)} \, dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{1-k x}{(1+(-2+k) x) \left((1-x) x (1-k x)\right)^{2/3}} \, dx$$

Optimal (type 3, 176 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-k x)}{(1-k)^{1/3}((1-x)\,x\,(1-k x))^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}(1-k)^{1/3}}+\frac{\text{Log}[1-(2-k)\,x]}{2^{2/3}(1-k)^{1/3}}+\frac{\text{Log}[1-k x]}{2\times 2^{2/3}(1-k)^{1/3}}-\frac{3\,\text{Log}\left[-1+k x+2^{2/3}(1-k)^{1/3}((1-x)\,x\,(1-k x))^{1/3}\right]}{2\times 2^{2/3}(1-k)^{1/3}}$$

Result (type 8, 35 leaves):

$$\int\frac{1-k x}{(1+(-2+k)\,x)\,((1-x)\,x\,(1-k x))^{2/3}}\,dx$$

Problem 46: Unable to integrate problem.

$$\int\frac{a+b x+c x^2}{(1-x+x^2)\,(1-x^3)^{1/3}}\,dx$$

Optimal (type 3, 493 leaves, 19 steps):

$$\frac{(a+b)\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{(a+b)\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{c\,\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{(a-c)\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{(b+c)\,\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+$$

$$\frac{(a+b)\,\text{Log}\left[(1-x)\,(1+x)^2\right]}{12\times 2^{1/3}}-\frac{(a-c)\,\text{Log}[1+x^3]}{6\times 2^{1/3}}-\frac{(b+c)\,\text{Log}[1+x^3]}{6\times 2^{1/3}}+\frac{(a+b)\,\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{(a+b)\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+$$

$$\frac{(b+c)\,\text{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}+\frac{(a-c)\,\text{Log}\left[-2^{1/3}x-(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}+\frac{1}{2}\,c\,\text{Log}\left[x+(1-x^3)^{1/3}\right]-\frac{(a+b)\,\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 8, 34 leaves):

$$\int\frac{a+b x+c x^2}{(1-x+x^2)\,(1-x^3)^{1/3}}\,dx$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(3-2x)^{11/2}\,(1+x+2x^2)^5}\,dx$$

Optimal (type 3, 407 leaves, 19 steps):

$$-\frac{19255}{395136\,(3-2x)^{9/2}}-\frac{462025}{30118144\,(3-2x)^{7/2}}-\frac{38491}{8605184\,(3-2x)^{5/2}}-\frac{141045}{120472576\,(3-2x)^{3/2}}-$$

$$\frac{38225}{240945152\,\sqrt{3-2x}}+\frac{x}{28\,(3-2x)^{9/2}\,(1+x+2x^2)^4}+\frac{23+73x}{1176\,(3-2x)^{9/2}\,(1+x+2x^2)^3}+\frac{1387+3049x}{32928\,(3-2x)^{9/2}\,(1+x+2x^2)^2}+$$

$$\frac{5\,(3049+4377x)}{153664\,(3-2x)^{9/2}\,(1+x+2x^2)}+\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\ \text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]}{3373232128}-$$

$$\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\ \text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]}{3373232128}+$$

$$\frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\ \text{Log}\left[3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\,\sqrt{3-2x}-2x\right]}{6746464256}-$$

$$\frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\ \text{Log}\left[3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\,\sqrt{3-2x}-2x\right]}{6746464256}$$

Result (type 3, 206 leaves) :

$$\frac{1}{30359089152}$$

$$\Bigg(-\Big(\Big(14\,\big(40289347-429812744x+135202154x^2-1073855156x^3+1627773523x^4-1470758860x^5+2888625656x^6-3106712560x^7+$$

$$2343370048x^8-2443779648x^9+1873554048x^{10}-677249280x^{11}+88070400x^{12}\big)\Big)\Big/\Big((3-2x)^{9/2}\,(1+x+2x^2)^4\Big)\Big)+$$

$$\frac{45\,i\left(53515\,i+284993\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{6-4x}}{\sqrt{-7-i\sqrt{7}}}\right]}{\sqrt{-\frac{1}{2}\,i\left(-7\,i+\sqrt{7}\right)}}-\frac{45\,i\left(-53515\,i+284993\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{6-4x}}{\sqrt{-7+i\sqrt{7}}}\right]}{\sqrt{\frac{1}{2}\,i\left(7\,i+\sqrt{7}\right)}}\Bigg)$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$$

Optimal (type 3, 648 leaves, 29 steps):

$$\frac{4718120139975}{351733660450816\,(3-2x)^{19/2}}-\frac{815900548375}{629418129227776\,(3-2x)^{17/2}}-\frac{3029508823715}{1555033025150976\,(3-2x)^{15/2}}-\frac{13515743021825}{13476952884641792\,(3-2x)^{13/2}}-$$
$$\frac{5846828446875}{14513641568075776\,(3-2x)^{11/2}}-\frac{37283626871975}{261245548225363968\,(3-2x)^{9/2}}-\frac{132355162272575}{2844673747342852096\,(3-2x)^{7/2}}-\frac{11557581705725}{812763927812243456\,(3-2x)^{5/2}}-$$
$$\frac{46601678385075}{11378694989371408384\,(3-2x)^{3/2}}-\frac{24229218097975}{22757389978742816768\,\sqrt{3-2x}}+\frac{x}{63\,(3-2x)^{19/2}\left(1+x+2x^2\right)^9}+\frac{53+173x}{7056\,(3-2x)^{19/2}\left(1+x+2x^2\right)^8}+$$
$$\frac{8477+21409x}{691488\,(3-2x)^{19/2}\left(1+x+2x^2\right)^7}+\frac{5\,(21409+47471x)}{6453888\,(3-2x)^{19/2}\left(1+x+2x^2\right)^6}+\frac{41\,(47471+92875x)}{90354432\,(3-2x)^{19/2}\left(1+x+2x^2\right)^5}+$$
$$\frac{41\,(3436375+5677637x)}{5059848192\,(3-2x)^{19/2}\left(1+x+2x^2\right)^4}+\frac{451\,(811091+998691x)}{10119696384\,(3-2x)^{19/2}\left(1+x+2x^2\right)^3}+\frac{451\,(28962039+14627273x)}{283351498752\,(3-2x)^{19/2}\left(1+x+2x^2\right)^2}+$$
$$\frac{11275\,(14627273-35058731x)}{3966920982528\,(3-2x)^{19/2}\left(1+x+2x^2\right)}+\frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]}{318603459702399434752}-$$
$$\frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]}{318603459702399434752}+$$
$$\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\,\text{Log}\left[3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\,\sqrt{3-2x}-2x\right]}{637206919404798869504}-$$
$$\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\,\text{Log}\left[3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\,\sqrt{3-2x}-2x\right]}{637206919404798869504}$$

Result (type 3, 662 leaves):

$$-\frac{47\sqrt{3-2x}-23(3-2x)^{3/2}}{4235364\left(14-7(3-2x)+(3-2x)^2\right)^9}-\frac{44193\sqrt{3-2x}-11993(3-2x)^{3/2}}{948721536\left(14-7(3-2x)+(3-2x)^2\right)^8}+$$

$$\frac{5\left(-1574149\sqrt{3-2x}+340449(3-2x)^{3/2}\right)}{185949421056\left(14-7(3-2x)+(3-2x)^2\right)^7}+\frac{5\left(-37938085\sqrt{3-2x}+5912661(3-2x)^{3/2}\right)}{10413167579136\left(14-7(3-2x)+(3-2x)^2\right)^6}-$$

$$\frac{5\left(107643741\sqrt{3-2x}+38010319(3-2x)^{3/2}\right)}{291568692215808\left(14-7(3-2x)+(3-2x)^2\right)^5}-\frac{-132204145097\sqrt{3-2x}+52802422641(3-2x)^{3/2}}{32655693528170496\left(14-7(3-2x)+(3-2x)^2\right)^4}-$$

$$\frac{-4402987778403\sqrt{3-2x}+1406968826615(3-2x)^{3/2}}{914359418788773888\left(14-7(3-2x)+(3-2x)^2\right)^3}-\frac{11\left(-6489356793153\sqrt{3-2x}+1953387138017(3-2x)^{3/2}\right)}{17068042484057112576\left(14-7(3-2x)+(3-2x)^2\right)^2}-$$

$$\frac{55\left(-4751425354423\sqrt{3-2x}+1410835658499(3-2x)^{3/2}\right)}{68272169936228450304\left(14-7(3-2x)+(3-2x)^2\right)}+\frac{1}{5367029731(3-2x)^{19/2}}+\frac{5}{4802079233(3-2x)^{17/2}}+$$

$$\frac{73}{23727920916(3-2x)^{15/2}}+\frac{165}{25705247659(3-2x)^{13/2}}+\frac{2365}{221460595216(3-2x)^{11/2}}+\frac{30349}{1993145356944(3-2x)^{9/2}}+$$

$$\frac{854095}{43406276662336(3-2x)^{7/2}}+\frac{75933}{3100448333024(3-2x)^{5/2}}+\frac{8519225}{260437659974016(3-2x)^{3/2}}+\frac{891605}{12401793332096\sqrt{3-2x}}-$$

$$\frac{11275\left(-34555708553\,i+2148932869\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7-i\sqrt{7}}}\right]}{22757389978742816768\sqrt{14\left(-7-i\sqrt{7}\right)}}-\frac{11275\left(34555708553\,i+2148932869\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7+i\sqrt{7}}}\right]}{22757389978742816768\sqrt{14\left(-7+i\sqrt{7}\right)}}$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx$$

Optimal (type 3, 1058 leaves, 49 steps):

$$-\frac{13056959628363355534285785425}{106924014357253562723941220352(3-2x)^{39/2}}-\frac{3948194343291401740321996415}{202881463139404195937734623232(3-2x)^{37/2}}-$$

$$\frac{304688229262620222736480811}{537361713180043545997243056128(3-2x)^{35/2}}+\frac{2124315846756567455653862925}{1688851098565851144562763890688(3-2x)^{33/2}}+$$

$$\frac{47657515074514118796095929535}{66632852434325399703658138959872(3-2x)^{31/2}}+\frac{34911619993974714062172751985}{124667917457770102671360389021696(3-2x)^{29/2}}+$$

$$\frac{149066309808794760843017404825}{1624981820656451683095663001731072\,(3-2x)^{27/2}}+\frac{15848613964169066543734380171}{601845118761648771516912222863360\,(3-2x)^{25/2}}+$$

$$\frac{11155168222970774232376891145}{1685166332532616560247354224017408\,(3-2x)^{23/2}}+\frac{14011818498091020272474956375}{10110997995195699361484125344104448\,(3-2x)^{21/2}}+$$

$$\frac{173441368149804378661935869705}{896508488907352010051592447177261056\,(3-2x)^{19/2}}-\frac{22724090823469905152713519545}{1604278348571050965355481221264572416\,(3-2x)^{17/2}}-$$

$$\frac{101190274412779618678573275245}{3963511214116714149701777134888943616\,(3-2x)^{15/2}}-\frac{460503190416958283087439337135}{34350430522344855964082068502370844672\,(3-2x)^{13/2}}-$$

$$\frac{2211619588790911794826342607495}{406920484649315986036049119181931544576\,(3-2x)^{11/2}}-\frac{143401467550777247627940437025}{73985542663511997461099839851260280832\,(3-2x)^{9/2}}-$$

$$\frac{4611053278117143010907562317585}{7250583181024175751187784305423507521536\,(3-2x)^{7/2}}-\frac{405965372440630510720926890227}{2071595194578335928910795515835287863296\,(3-2x)^{5/2}}-$$

$$\frac{4986681479187781853417316522775}{87006998172290109014253411665082090258432\,(3-2x)^{3/2}}-\frac{927027754781476746208047620505}{58004665448193406009502274443388060172288\,\sqrt{3-2x}}+$$

$$\frac{x}{133\,(3-2x)^{39/2}\,(1+x+2x^2)^{19}}+\frac{113+373x}{33516\,(3-2x)^{39/2}\,(1+x+2x^2)^{18}}+\frac{40657+107329x}{7976808\,(3-2x)^{39/2}\,(1+x+2x^2)^{17}}+\frac{5\,(751303+1831285x)}{595601664\,(3-2x)^{39/2}\,(1+x+2x^2)^{16}}+$$

$$\frac{184959785+429411497x}{25015269888\,(3-2x)^{39/2}\,(1+x+2x^2)^{15}}+\frac{41652915209+92630823167x}{4902992898048\,(3-2x)^{39/2}\,(1+x+2x^2)^{14}}+\frac{2871555518177+6100156355517x}{297448235814912\,(3-2x)^{39/2}\,(1+x+2x^2)^{13}}+$$

$$\frac{77559130805859+156274047129113x}{7138757659557888\,(3-2x)^{39/2}\,(1+x+2x^2)^{12}}+\frac{5\,(2656658801194921+5020880176134289x)}{1099368679571914752\,(3-2x)^{39/2}\,(1+x+2x^2)^{11}}+$$

$$\frac{45187921585208601+78752911037377255x}{3420258114223734784\,(3-2x)^{39/2}\,(1+x+2x^2)^{10}}+\frac{6063974149878048635+9477172618423641847x}{430952522392190582784\,(3-2x)^{39/2}\,(1+x+2x^2)^{9}}+$$

$$\frac{691833601144925854831+919498192874055581221x}{48266682507925345271808\,(3-2x)^{39/2}\,(1+x+2x^2)^{8}}+\frac{23\,(919498192874055581221+908287136092467468517x)}{1576711628592227945545728\,(3-2x)^{39/2}\,(1+x+2x^2)^{7}}+$$

$$\frac{115\,(908287136092467468517+298281884944522225747x)}{10187982830903626725064704\,(3-2x)^{39/2}\,(1+x+2x^2)^{6}}+\frac{23\,(2599313568802265110081-10426142448623187379187x)}{20375965661807253450129408\,(3-2x)^{39/2}\,(1+x+2x^2)^{5}}-$$

$$\frac{23\,(10426142448623187379187+27513723463194262383705x)}{20018492580021161284337664\,(3-2x)^{39/2}\,(1+x+2x^2)^{4}}-\frac{115\,(26513224428169016478843+30673415406553789342019x)}{76434244396444433994743808\,(3-2x)^{39/2}\,(1+x+2x^2)^{3}}-$$

$$\frac{115\,(88411609113007981044643-5712269536245152162963x)}{125891696652967303050166272\,(3-2x)^{39/2}\,(1+x+2x^2)^{2}}-\frac{115\,(28561347681225760814815+965934812839019490346107x)}{195831528126838026966925312\,(3-2x)^{39/2}\,(1+x+2x^2)}+$$

$$\left(115\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(30\,297\,118\,912\,219\,360\,725\,028\,693\,061+8\,061\,110\,911\,143\,276\,053\,983\,022\,787\sqrt{14}\right)\text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]\right)\Big/$$
$$812\,065\,316\,274\,707\,684\,133\,031\,842\,207\,432\,842\,412\,032-$$
$$\left(115\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(30\,297\,118\,912\,219\,360\,725\,028\,693\,061+8\,061\,110\,911\,143\,276\,053\,983\,022\,787\sqrt{14}\right)\text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]\right)\Big/$$
$$812\,065\,316\,274\,707\,684\,133\,031\,842\,207\,432\,842\,412\,032+\left(115\left(30\,297\,118\,912\,219\,360\,725\,028\,693\,061-8\,061\,110\,911\,143\,276\,053\,983\,022\,787\sqrt{14}\right)\right.$$
$$\left.\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\,\text{Log}\left[3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right]\right)\Big/1\,624\,130\,632\,549\,415\,368\,266\,063\,684\,414\,865\,684\,824\,064-$$
$$\left(115\left(30\,297\,118\,912\,219\,360\,725\,028\,693\,061-8\,061\,110\,911\,143\,276\,053\,983\,022\,787\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\right.$$
$$\left.\text{Log}\left[3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right]\right)\Big/1\,624\,130\,632\,549\,415\,368\,266\,063\,684\,414\,865\,684\,824\,064$$

Result (type 3, 1242 leaves) :

$$-\frac{393\sqrt{3-2x}+287(3-2x)^{3/2}}{150\,276\,832\,468\left(14-7(3-2x)+(3-2x)^2\right)^{19}}-\frac{-4\,226\,921\sqrt{3-2x}+1\,313\,129(3-2x)^{3/2}}{75\,739\,523\,563\,872\left(14-7(3-2x)+(3-2x)^2\right)^{18}}-$$
$$\frac{-3\,401\,932\,701\sqrt{3-2x}+760\,755\,809(3-2x)^{3/2}}{36\,052\,013\,216\,403\,072\left(14-7(3-2x)+(3-2x)^2\right)^{17}}-\frac{5\left(-146\,490\,500\,023\sqrt{3-2x}+16\,144\,709\,919(3-2x)^{3/2}\right)}{16\,151\,301\,920\,948\,576\,256\left(14-7(3-2x)+(3-2x)^2\right)^{16}}-$$
$$\frac{9\,745\,709\,632\,283\sqrt{3-2x}-4\,557\,912\,048\,927(3-2x)^{3/2}}{452\,236\,453\,786\,560\,135\,168\left(14-7(3-2x)+(3-2x)^2\right)^{15}}-\frac{435\,856\,117\,815\,771\sqrt{3-2x}-123\,609\,208\,162\,571(3-2x)^{3/2}}{9\,330\,352\,099\,175\,345\,946\,624\left(14-7(3-2x)+(3-2x)^2\right)^{14}}-$$
$$\frac{127\,435\,522\,656\,997\,631\sqrt{3-2x}-31\,270\,302\,414\,674\,811(3-2x)^{3/2}}{3\,396\,248\,164\,099\,825\,924\,571\,136\left(14-7(3-2x)+(3-2x)^2\right)^{13}}+\frac{5\left(-1\,540\,359\,167\,602\,841\,319\sqrt{3-2x}+342\,026\,557\,757\,088\,031(3-2x)^{3/2}\right)}{380\,379\,794\,379\,180\,503\,551\,967\,232\left(14-7(3-2x)+(3-2x)^2\right)^{12}}+$$
$$\frac{5\left(-21\,084\,628\,139\,481\,190\,687\sqrt{3-2x}+4\,158\,669\,924\,550\,257\,827(3-2x)^{3/2}\right)}{13\,017\,441\,852\,087\,510\,566\,000\,656\,384\left(14-7(3-2x)+(3-2x)^2\right)^{11}}-$$

$$\frac{1\,633\,293\,973\,597\,342\,712\,581\,\sqrt{3-2\,x}-237\,080\,744\,154\,193\,384\,005\,(3-2\,x)^{3/2}}{728\,976\,743\,716\,900\,591\,696\,036\,757\,504\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{10}}-$$

$$\frac{7\,350\,432\,513\,431\,022\,017\,155\,\sqrt{3-2\,x}+5\,131\,564\,318\,471\,376\,538\,977\,(3-2\,x)^{3/2}}{61\,234\,046\,472\,219\,649\,702\,467\,087\,630\,336\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{9}}-$$

$$\frac{-113\,207\,386\,492\,327\,172\,550\,771\,\sqrt{3-2\,x}+43\,421\,160\,367\,342\,900\,895\,387\,(3-2\,x)^{3/2}}{279\,927\,069\,587\,289\,827\,211\,278\,114\,881\,536\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{8}}-$$

$$\frac{-22\,463\,796\,720\,502\,183\,624\,842\,107\,\sqrt{3-2\,x}+7\,094\,978\,194\,424\,786\,431\,173\,663\,(3-2\,x)^{3/2}}{54\,865\,705\,639\,108\,806\,133\,410\,510\,516\,781\,056\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{7}}-$$

$$\frac{5\,\left(-186\,257\,412\,289\,925\,530\,757\,362\,143\,\sqrt{3-2\,x}+55\,540\,178\,588\,722\,046\,667\,113\,711\,(3-2\,x)^{3/2}\right)}{3\,072\,479\,515\,790\,093\,143\,470\,988\,588\,939\,739\,136\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{6}}-$$

$$\frac{23\,\left(-255\,056\,047\,077\,847\,659\,080\,618\,951\,\sqrt{3-2\,x}+74\,443\,988\,473\,272\,328\,189\,316\,355\,(3-2\,x)^{3/2}\right)}{28\,676\,475\,480\,707\,536\,005\,729\,226\,830\,104\,231\,936\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{5}}-$$

$$\frac{23\,\left(-1\,110\,057\,788\,286\,806\,589\,656\,260\,577\,\sqrt{3-2\,x}+321\,533\,953\,909\,984\,640\,923\,113\,289\,(3-2\,x)^{3/2}\right)}{188\,927\,367\,872\,896\,707\,802\,451\,376\,763\,039\,645\,696\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{4}}-$$

$$\frac{23\,\left(-4\,820\,387\,670\,797\,872\,511\,726\,954\,245\,\sqrt{3-2\,x}+1\,394\,304\,490\,531\,377\,203\,111\,252\,689\,(3-2\,x)^{3/2}\right)}{1\,220\,761\,453\,947\,947\,958\,108\,147\,357\,545\,794\,633\,728\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{3}}-$$

$$\frac{23\,\left(-17\,490\,402\,570\,151\,108\,581\,128\,226\,213\,\sqrt{3-2\,x}+5\,072\,167\,085\,782\,230\,110\,284\,731\,077\,(3-2\,x)^{3/2}\right)}{6\,214\,785\,583\,735\,007\,786\,732\,386\,547\,505\,863\,589\,888\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)^{2}}-$$

$$\frac{115\,\left(-82\,782\,386\,138\,609\,724\,168\,863\,115\,877\,\sqrt{3-2\,x}+24\,217\,623\,575\,858\,523\,510\,208\,130\,121\,(3-2\,x)^{3/2}\right)}{174\,013\,996\,344\,580\,218\,028\,506\,823\,330\,164\,180\,516\,864\,\left(14-7\,(3-2\,x)+(3-2\,x)^2\right)}+$$

$$\frac{1}{3\,111\,898\,385\,606\,868\,039\,(3-2\,x)^{39/2}}+\frac{10}{2\,952\,313\,853\,011\,644\,037\,(3-2\,x)^{37/2}}+\frac{143}{7\,819\,642\,097\,165\,976\,098\,(3-2\,x)^{35/2}}+$$

$$\frac{355}{5\,266\,289\,575\,642\,392\,066\,(3-2\,x)^{33/2}}+\frac{52\,865}{277\,038\,748\,585\,308\,867\,472\,(3-2\,x)^{31/2}}+\frac{14\,333}{32\,395\,660\,116\,830\,472\,406\,(3-2\,x)^{29/2}}+$$

$$\frac{1\,478\,345}{1\,689\,042\,692\,987\,850\,837\,168\,(3-2\,x)^{27/2}}+\frac{475\,387}{312\,785\,683\,886\,639\,043\,920\,(3-2\,x)^{25/2}}+\frac{16\,575\,515}{7\,006\,399\,319\,060\,714\,583\,808\,(3-2\,x)^{23/2}}+$$

$$\frac{246866015}{73567192850137503129984\,(3-2x)^{21/2}}+\frac{8192823353}{1863702218870150079292928\,(3-2x)^{19/2}}+\frac{8972680075}{1667523037936450070946304\,(3-2x)^{17/2}}+\frac{102495360575}{16479051198430800701116416\,(3-2x)^{15/2}}+\frac{122484655975}{17852305464966700759542784\,(3-2x)^{13/2}}+\frac{10815878546425}{1480368099325700262983624704\,(3-2x)^{11/2}}+\frac{769045155125}{100934188590388654294338048\,(3-2x)^{9/2}}+\frac{838467657280275}{105509871806486273289014706176\,(3-2x)^{7/2}}+\frac{9270470094105}{1076631344964145645806272512\,(3-2x)^{5/2}}+\frac{320421783064625}{30145677658996078082575630336\,(3-2x)^{3/2}}+\frac{683151246370725}{30145677658996078082575630336\,\sqrt{3-2x}}-$$

$$\left(115\left(-117022014202441653827938545631\,i+8061110911143276053983022787\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7-i\sqrt{7}}}\right]\right)\Big/$$

$$\left(58004665448193406009502274443388060172288\sqrt{14\left(-7-i\sqrt{7}\right)}\right)-$$

$$\left(115\left(117022014202441653827938545631\,i+8061110911143276053983022787\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7+i\sqrt{7}}}\right]\right)\Big/$$

$$\left(58004665448193406009502274443388060172288\sqrt{14\left(-7+i\sqrt{7}\right)}\right)$$

Problem 50: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx$$

Optimal (type 3, 378 leaves, 14 steps):

$$-\frac{3450497-2004270\,x}{123480000\,\left(3-2\,x+x^2\right)^{9/2}}-\frac{4878869-2578034\,x}{411600000\,\left(3-2\,x+x^2\right)^{7/2}}-\frac{30316369-15043110\,x}{6860000000\,\left(3-2\,x+x^2\right)^{5/2}}-\frac{63043297-29625922\,x}{41160000000\,\left(3-2\,x+x^2\right)^{3/2}}-$$

$$\frac{31\,\left(7434109-3088870\,x\right)}{411600000000\,\sqrt{3-2\,x+x^2}}-\frac{1-10\,x}{280\,\left(3-2\,x+x^2\right)^{9/2}\,\left(1+x+2\,x^2\right)^4}+\frac{28+67\,x}{1050\,\left(3-2\,x+x^2\right)^{9/2}\,\left(1+x+2\,x^2\right)^3}+\frac{5485+8878\,x}{117600\,\left(3-2\,x+x^2\right)^{9/2}\,\left(1+x+2\,x^2\right)^2}+$$

$$\frac{3\,\left(8822+8233\,x\right)}{343000\,\left(3-2\,x+x^2\right)^{9/2}\,\left(1+x+2\,x^2\right)}+\frac{1}{137200000000}\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\,\sqrt{2}\right)}\;\text{ArcTan}\Big[\frac{1}{\sqrt{3-2\,x+x^2}}$$

$$\sqrt{\frac{5}{7\,\left(151363871237318045+110320475741093888\,\sqrt{2}\right)}}\;\left(308108167+312239803\,\sqrt{2}+\left(932587773+620347970\,\sqrt{2}\right)\,x\right)\Big]-$$

$$\frac{1}{137200000000}\sqrt{\frac{1}{70}\left(-151363871237318045+110320475741093888\,\sqrt{2}\right)}\;\text{ArcTanh}\Big[\frac{1}{\sqrt{3-2\,x+x^2}}$$

$$\sqrt{\frac{5}{7\,\left(-151363871237318045+110320475741093888\,\sqrt{2}\right)}}\;\left(308108167-312239803\,\sqrt{2}+\left(932587773-620347970\,\sqrt{2}\right)\,x\right)\Big]$$

Result (type 3, 1236 leaves) :

$$\sqrt{3-2\,x+x^2}\,\left(\frac{1}{225000\,\left(3-2\,x+x^2\right)^5}+\frac{1+2\,x}{350000\,\left(3-2\,x+x^2\right)^4}+\frac{3\,\left(-38+45\,x\right)}{8750000\,\left(3-2\,x+x^2\right)^3}+\frac{-2003+1198\,x}{52500000\,\left(3-2\,x+x^2\right)^2}+\frac{-97229+29420\,x}{1050000000\,\left(3-2\,x+x^2\right)}+\right.$$

$$\left.\frac{-797-1998\,x}{28000000\,\left(1+x+2\,x^2\right)^4}+\frac{-14087-5995\,x}{105000000\,\left(1+x+2\,x^2\right)^3}+\frac{-795589+1892994\,x}{11760000000\,\left(1+x+2\,x^2\right)^2}+\frac{3035369+14037055\,x}{34300000000\,\left(1+x+2\,x^2\right)}\right)+$$

$$\frac{1}{68600000000\,\sqrt{70\,\left(-5+i\,\sqrt{7}\right)}}\left(310173985\,i+44900803\,\sqrt{7}\right)$$

$$\text{ArcTan}\Big[\Big(9627448535205165+357977536529228045\,i\,\sqrt{7}-2892591314086740000\,x+36106220736881480\,i\,\sqrt{7}\,x+464983088285203040\,x^2-$$

$$1038569725622524380\,i\,\sqrt{7}\,x^2+12836598046940220\,x^3+328748064746064540\,i\,\sqrt{7}\,x^3-487447134867348425\,x^4-$$

$$428071291440525685\,i\,\sqrt{7}\,x^4+358541546158555136\,i\,\sqrt{10\,\left(-5+i\,\sqrt{7}\right)}\,\sqrt{3-2\,x+x^2}+220640951482187776\,i\,\sqrt{10\,\left(-5+i\,\sqrt{7}\right)}\,x$$

$$\sqrt{3-2\,x+x^2}+579182497640742912\,i\,\sqrt{10\,\left(-5+i\,\sqrt{7}\right)}\,x^2\,\sqrt{3-2\,x+x^2}-275801189352734720\,i\,\sqrt{10\,\left(-5+i\,\sqrt{7}\right)}\,x^3\,\sqrt{3-2\,x+x^2}\Big)\Big/$$

$$\Big(4321741285513437647\,i+827387564543169945\,\sqrt{7}+3694994885631086104\,i\,x+285423303382928480\,\sqrt{7}\,x+$$

$$5471192788852131980\,i\,x^2-70525532316488480\,\sqrt{7}\,x^2-6268363351511187532\,i\,x^3+$$

$$137879256656321740\sqrt{7}\,x^3 + 2092254277956040633\,i\,x^4 + 70562873851568315\sqrt{7}\,x^4\Big)\Big] -$$

$$\frac{1}{68600000000\sqrt{70\left(5 + i\sqrt{7}\right)}}\,i\left(-310173985\,i + 44900803\sqrt{7}\right)\text{ArcTan}\Big[\Big(35\,\Big(15210275631276955\,i + 23639644701233427\sqrt{7} -$$

$$80355173705781000\,i\,x + 8154951525226528\sqrt{7}\,x + 32801021588957180\,i\,x^2 - 2015015209042528\sqrt{7}\,x^2 -$$

$$22632774169109180\,i\,x^3 + 3939407333037764\sqrt{7}\,x^3 - 9346476174243955\,i\,x^4 + 2016082110044809\sqrt{7}\,x^4\Big)\Big)\Big/$$

$$\Big(-9627448535205165 + 357977536529228045\,i\sqrt{7} + 2892591314086740000\,x + 36106220736881480\,i\sqrt{7}\,x -$$

$$464983088285203040\,x^2 - 1038569725622524380\,i\sqrt{7}\,x^2 - 12836598046940220\,x^3 + 328748064746064540\,i\sqrt{7}\,x^3 +$$

$$487447134867348425\,x^4 - 428071291440525685\,i\sqrt{7}\,x^4 - 27580118935273472\,i\sqrt{70\left(5 + i\sqrt{7}\right)}\,\sqrt{3 - 2x + x^2} -$$

$$27580118935273472\,i\sqrt{70\left(5 + i\sqrt{7}\right)}\,x^2\sqrt{3 - 2x + x^2} + 55160237870546944\,i\sqrt{70\left(5 + i\sqrt{7}\right)}\,x^3\sqrt{3 - 2x + x^2}\Big)\Big] -$$

$$\frac{\left(-310173985\,i + 44900803\sqrt{7}\right)\text{Log}\Big[\left(-i + \sqrt{7} - 4\,i\,x\right)^2\left(i + \sqrt{7} + 4\,i\,x\right)^2\Big]}{137200000000\sqrt{70\left(5 + i\sqrt{7}\right)}} +$$

$$\frac{i\left(310173985\,i + 44900803\sqrt{7}\right)\text{Log}\Big[\left(-i + \sqrt{7} - 4\,i\,x\right)^2\left(i + \sqrt{7} + 4\,i\,x\right)^2\Big]}{137200000000\sqrt{70\left(-5 + i\sqrt{7}\right)}} -$$

$$\Big(i\left(310173985\,i + 44900803\sqrt{7}\right)\text{Log}\Big[\left(1 + x + 2x^2\right)$$

$$\Big(-13\,i + 15\sqrt{7} + 22\,i\,x - 10\sqrt{7}\,x + 9\,i\,x^2 + 5\sqrt{7}\,x^2 + i\sqrt{70\left(-5 + i\sqrt{7}\right)}\,\sqrt{3 - 2x + x^2} - i\sqrt{70\left(-5 + i\sqrt{7}\right)}\,x\sqrt{3 - 2x + x^2}\Big)\Big]\Big)\Big/$$

$$\left(137200000000\sqrt{70\left(-5 + i\sqrt{7}\right)}\right) + \Big(\left(-310173985\,i + 44900803\sqrt{7}\right)\text{Log}\Big[\left(1 + x + 2x^2\right)\Big(-163\,i + 15\sqrt{7} + 122\,i\,x - 10\sqrt{7}\,x -$$

$$41\,i\,x^2 + 5\sqrt{7}\,x^2 - 13\,i\sqrt{10\left(5 + i\sqrt{7}\right)}\,\sqrt{3 - 2x + x^2} + 5\,i\sqrt{10\left(5 + i\sqrt{7}\right)}\,x\sqrt{3 - 2x + x^2}\Big)\Big]\Big)\Big/\left(137200000000\sqrt{70\left(5 + i\sqrt{7}\right)}\right)$$

Problem 51: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(3-2\,x+x^2\right)^{21/2}\,\left(1+x+2\,x^2\right)^{10}}\,dx$$

Optimal (type 3, 638 leaves, 24 steps) :

$$\frac{37358055634422583-14024622879097678\,x}{1840124479200000000\,\left(3-2\,x+x^2\right)^{19/2}}+\frac{476849951294984711-125181871472148210\,x}{104273720488000000000\,\left(3-2\,x+x^2\right)^{17/2}}+$$

$$\frac{7851758375483333511+1942164996204584234\,x}{15641058073200000000000\,\left(3-2\,x+x^2\right)^{15/2}}-\frac{11\,\left(7502325106308201089-7813986379726516886\,x\right)}{406667509903200000000000\,\left(3-2\,x+x^2\right)^{13/2}}-$$

$$\frac{3\,\left(69053268515296359011-44840736195018286006\,x\right)}{1147010925368000000000000\,\left(3-2\,x+x^2\right)^{11/2}}-\frac{838519439380295335657-466189390555853643870\,x}{9384634843920000000000000\,\left(3-2\,x+x^2\right)^{9/2}}-$$

$$\frac{1117646664729238460189-568839749685437871554\,x}{31282116146400000000000000\,\left(3-2\,x+x^2\right)^{7/2}}-\frac{6551405511565449301689-3127298559983309301910\,x}{521368602440000000000000000\,\left(3-2\,x+x^2\right)^{5/2}}-$$

$$\frac{4179039782398459850819-1886993445589652402694\,x}{1042737204880000000000000000\,\left(3-2\,x+x^2\right)^{3/2}}-\frac{12105495874518671061833-5117656435043679338190\,x}{10427372048800000000000000000\,\sqrt{3-2\,x+x^2}}-$$

$$\frac{1-10\,x}{630\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)^9}+\frac{887+2218\,x}{88200\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)^8}+\frac{14453+29371\,x}{1080450\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)^7}+$$

$$\frac{8837931+17459234\,x}{605052000\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)^6}+\frac{447940041+813432205\,x}{26471025000\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)^5}+$$

$$\frac{592729157441+911061463974\,x}{29647548000000\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)^4}+\frac{277010166219+310705340015\,x}{12353145000000\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)^3}+$$

$$\frac{5488221294349+1384103301166\,x}{276710448000000\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)^2}-\frac{37857197792117+146548895467025\,x}{2421216420000000\,\left(3-2\,x+x^2\right)^{19/2}\,\left(1+x+2\,x^2\right)}+\frac{1}{32282885600000000000000}$$

$$\sqrt{\left(\frac{1}{70}\left(81042225921274689605478944797800854846405+57305922523001707126026363878666500308992\,\sqrt{2}\right)\right)}$$

$$\text{ArcTan}\Big[\frac{1}{\sqrt{3-2\,x+x^2}}\sqrt{\left(5\Big/\left(7\left(81042225921274689605478944797800854846405+57305922523001707126026363878666500308992\,\sqrt{2}\right)\right)\right)}$$

$$\left(272944589523248381749+191941026386645109841\,\sqrt{2}+\right.$$

$$\left.\left(656826642296538601431+464885615909893491590\,\sqrt{2}\right)\,x\right)\Big]-\frac{1}{32282885600000000000000}$$

$$\sqrt{\left(\frac{1}{70}\left(-81042225921274689605478944797800854846405+57305922523001707126026363878666500308992\,\sqrt{2}\right)\right)}\ \text{ArcTanh}\Big[$$

$$\frac{1}{\sqrt{3-2\,x+x^2}}\sqrt{\left(5\Big/\left(7\left(-81042225921274689605478944797800854846405+57305922523001707126026363878666500308992\,\sqrt{2}\right)\right)\right)}$$

$$\left(272944589523248381749-191941026386645109841\,\sqrt{2}+\left(656826642296538601431-464885615909893491590\,\sqrt{2}\right)\,x\right)\Big]$$

Result (type 3, 1431 leaves):

$$\sqrt{3-2\,x+x^2}\,\left(\frac{1-x}{11875000000\,\left(3-2\,x+x^2\right)^{10}}+\frac{265-113\,x}{403750000000\,\left(3-2\,x+x^2\right)^9}+\frac{82361-4841\,x}{60562500000000\,\left(3-2\,x+x^2\right)^8}+\right.$$

$$\frac{1 062 937 + 1 642 511 x}{1 574 625 000 000 000 \left(3 - 2 x + x^2\right)^7} + \frac{7 \left(- 678 331 + 833 371 x\right)}{2 220 625 000 000 000 \left(3 - 2 x + x^2\right)^6} + \frac{7 \left(- 73 161 291 + 43 964 675 x\right)}{90 843 750 000 000 000 \left(3 - 2 x + x^2\right)^5} +$$

$$\frac{- 1 340 879 383 + 430 593 031 x}{181 687 500 000 000 000 \left(3 - 2 x + x^2\right)^4} - \frac{11 \left(1 626 125 723 + 112 950 205 x\right)}{3 028 125 000 000 000 000 \left(3 - 2 x + x^2\right)^3} - \frac{11 \left(3 311 570 647 + 15 286 717 673 x\right)}{36 337 500 000 000 000 000 \left(3 - 2 x + x^2\right)^2} -$$

$$\frac{11 \left(- 411 521 923 277 + 484 788 625 685 x\right)}{363 375 000 000 000 000 000 \left(3 - 2 x + x^2\right)} + \frac{251 943 + 221 770 x}{6 300 000 000 000 \left(1 + x + 2 x^2\right)^9} - \frac{73 \left(- 888 423 + 1 604 678 x\right)}{882 000 000 000 000 \left(1 + x + 2 x^2\right)^8} +$$

$$\frac{- 2 596 903 794 - 4 965 311 863 x}{10 804 500 000 000 000 \left(1 + x + 2 x^2\right)^7} + \frac{- 539 608 494 637 - 334 647 150 510 x}{1 210 104 000 000 000 000 \left(1 + x + 2 x^2\right)^6} + \frac{- 40 800 462 989 458 + 56 711 874 696 335 x}{264 710 250 000 000 000 000 \left(1 + x + 2 x^2\right)^5} +$$

$$\frac{42 018 358 198 215 561 + 129 196 597 088 670 934 x}{296 475 480 000 000 000 000 000 \left(1 + x + 2 x^2\right)^4} + \frac{62 819 559 864 314 747 + 169 630 389 653 846 945 x}{370 594 350 000 000 000 000 000 \left(1 + x + 2 x^2\right)^3} +$$

$$\left.\frac{1 082 422 109 196 374 795 + 4 797 048 907 791 526 114 x}{8 301 313 440 000 000 000 000 000 \left(1 + x + 2 x^2\right)^2} + \frac{65 571 203 144 429 922 747 + 367 152 793 968 978 953 465 x}{363 182 463 000 000 000 000 000 000 \left(1 + x + 2 x^2\right)}\right) +$$

$$\frac{1}{16 141 442 800 000 000 000 000 000 \sqrt{70 \left(- 5 + \mathrm{i} \sqrt{7}\right)}} \left(232 442 807 954 946 745 795 \mathrm{i} + 21 634 177 831 191 924 841 \sqrt{7}\right)$$

$$\mathrm{ArcTan}\Big[\Big(- 135 063 738 860 435 016 899 586 558 948 733 259 113 515 + 188 630 894 626 466 690 216 855 285 995 045 889 396 405 \mathrm{i} \sqrt{7} -$$

$$1 506 241 361 872 688 008 559 268 776 761 430 483 700 000 x - 105 711 500 937 472 192 718 115 651 350 352 447 938 680 \mathrm{i} \sqrt{7} x +$$

$$491 153 540 508 443 587 025 809 789 813 541 985 707 360 x^2 - 460 764 064 177 139 993 399 975 100 872 663 310 399 420 \mathrm{i} \sqrt{7} x^2 -$$

$$180 084 985 147 246 689 199 448 745 264 977 678 818 020 x^3 + 197 868 296 377 913 870 863 837 680 953 446 009 396 860 \mathrm{i} \sqrt{7} x^3 -$$

$$176 004 816 500 761 880 926 774 485 599 831 047 775 825 x^4 - 207 342 833 228 459 577 163 557 043 035 558 264 835 165 \mathrm{i} \sqrt{7} x^4 +$$

$$186 244 248 199 755 548 159 585 682 605 666 126 004 224 \mathrm{i} \sqrt{10 \left(- 5 + \mathrm{i} \sqrt{7}\right)} \sqrt{3 - 2 x + x^2} +$$

$$114 611 845 046 003 414 252 052 727 757 333 000 617 984 \mathrm{i} \sqrt{10 \left(- 5 + \mathrm{i} \sqrt{7}\right)} x \sqrt{3 - 2 x + x^2} +$$

$$300 856 093 245 758 962 411 638 410 362 999 126 622 208 \mathrm{i} \sqrt{10 \left(- 5 + \mathrm{i} \sqrt{7}\right)} x^2 \sqrt{3 - 2 x + x^2} -$$

$$143 264 806 307 504 267 815 065 909 696 666 250 772 480 \mathrm{i} \sqrt{10 \left(- 5 + \mathrm{i} \sqrt{7}\right)} x^3 \sqrt{3 - 2 x + x^2}\Big) \Big/$$

$$\Big(2 368 773 290 838 836 979 864 678 493 023 884 746 594 823 \mathrm{i} + 423 642 940 259 238 735 473 942 663 180 025 956 729 505 \sqrt{7} +$$

$$1 890 613 486 065 620 301 760 074 218 556 745 311 646 936 \mathrm{i} x + 6 150 574 559 311 228 258 394 328 777 942 059 796 320 \sqrt{7} x +$$

$$2 511 300 259 855 822 962 340 893 027 852 239 157 667 820 \mathrm{i} x^2 - 2 027 867 550 801 106 189 867 763 431 094 227 596 320 \sqrt{7} x^2 -$$

$$3 134 217 746 230 760 357 128 318 797 499 380 812 303 788 \mathrm{i} x^3 + 63 430 431 602 720 043 279 192 866 968 369 397 935 660 \sqrt{7} x^3 +$$

$944\,749\,064\,886\,626\,467\,328\,385\,369\,190\,460\,703\,669\,697\,i\,x^4 + 16\,381\,317\,765\,107\,264\,789\,462\,917\,221\,030\,750\,634\,835\,\sqrt{7}\,x^4\big)\big] -$

$\frac{1}{16\,141\,442\,800\,000\,000\,000\,000\,000\,\sqrt{70\,(5 + i\,\sqrt{7})}}\,i\,\left(-232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\right)$

$\text{ArcTan}\Big[\Big(35\,\big(4\,362\,494\,290\,663\,946\,676\,585\,186\,218\,212\,607\,628\,595\,i + 12\,104\,084\,007\,406\,821\,013\,541\,218\,948\,000\,741\,620\,843\,\sqrt{7} -$
$40\,919\,031\,596\,617\,332\,707\,196\,094\,500\,783\,237\,405\,000\,i\,x + 175\,730\,701\,694\,606\,521\,668\,409\,393\,655\,487\,422\,752\,\sqrt{7}\,x +$
$26\,487\,288\,329\,265\,127\,577\,733\,965\,853\,364\,310\,310\,620\,i\,x^2 - 57\,939\,072\,880\,031\,605\,424\,793\,240\,888\,406\,502\,752\,\sqrt{7}\,x^2 -$
$15\,238\,894\,149\,752\,825\,683\,924\,814\,021\,007\,863\,070\,620\,i\,x^3 + 1\,812\,298\,045\,792\,001\,236\,548\,367\,627\,667\,697\,083\,876\,\sqrt{7}\,x^3 -$
$795\,837\,271\,959\,975\,808\,913\,244\,203\,765\,619\,963\,595\,i\,x^4 + 468\,037\,650\,431\,636\,136\,841\,797\,634\,886\,592\,875\,281\,\sqrt{7}\,x^4\big)\Big) \Big/$

$\Big(135\,063\,738\,860\,435\,016\,899\,586\,558\,948\,733\,259\,113\,515 + 188\,630\,894\,626\,466\,690\,216\,855\,285\,995\,045\,889\,396\,405\,i\,\sqrt{7} +$
$1\,506\,241\,361\,872\,688\,008\,559\,268\,776\,761\,430\,483\,700\,000\,x - 105\,711\,500\,937\,472\,192\,718\,115\,651\,350\,352\,447\,938\,680\,i\,\sqrt{7}\,x -$
$491\,153\,540\,508\,443\,587\,025\,809\,789\,813\,541\,985\,707\,360\,x^2 - 460\,764\,064\,177\,139\,993\,399\,975\,100\,872\,663\,310\,399\,420\,i\,\sqrt{7}\,x^2 +$
$180\,084\,985\,147\,246\,689\,199\,448\,745\,264\,977\,678\,818\,020\,x^3 + 197\,868\,296\,377\,913\,870\,863\,837\,680\,953\,446\,009\,396\,860\,i\,\sqrt{7}\,x^3 +$
$176\,004\,816\,500\,761\,880\,926\,774\,485\,599\,831\,047\,775\,825\,x^4 - 207\,342\,833\,228\,459\,577\,163\,557\,043\,035\,558\,264\,835\,165\,i\,\sqrt{7}\,x^4 -$
$14\,326\,480\,630\,750\,426\,781\,506\,590\,969\,666\,625\,077\,248\,i\,\sqrt{70\,(5 + i\,\sqrt{7})}\,\sqrt{3 - 2\,x + x^2} - 14\,326\,480\,630\,750\,426\,781\,506\,590\,969\,666\,625\,077\,248$
$i\,\sqrt{70\,(5 + i\,\sqrt{7})}\,x^2\,\sqrt{3 - 2\,x + x^2} + 28\,652\,961\,261\,500\,853\,563\,013\,181\,939\,333\,250\,154\,496\,i\,\sqrt{70\,(5 + i\,\sqrt{7})}\,x^3\,\sqrt{3 - 2\,x + x^2}\Big)\Big] -$

$\Big(\big(-232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\big)\,\text{Log}\Big[\big(-i + \sqrt{7} - 4\,i\,x\big)^2\,\big(i + \sqrt{7} + 4\,i\,x\big)^2\Big]\Big) \Big/$
$\Big(32\,282\,885\,600\,000\,000\,000\,000\,000\,\sqrt{70\,(5 + i\,\sqrt{7})}\Big) +$

$\Big(i\,\big(232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\big)\,\text{Log}\Big[\big(-i + \sqrt{7} - 4\,i\,x\big)^2\,\big(i + \sqrt{7} + 4\,i\,x\big)^2\Big]\Big) \Big/$
$\Big(32\,282\,885\,600\,000\,000\,000\,000\,000\,\sqrt{70\,(-5 + i\,\sqrt{7})}\Big) -$

$\Big(i\,\big(232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\big)\,\text{Log}\Big[\big(1 + x + 2\,x^2\big)$
$\Big(-13\,i + 15\,\sqrt{7} + 22\,i\,x - 10\,\sqrt{7}\,x + 9\,i\,x^2 + 5\,\sqrt{7}\,x^2 + i\,\sqrt{70\,(-5 + i\,\sqrt{7})}\,\sqrt{3 - 2\,x + x^2} - i\,\sqrt{70\,(-5 + i\,\sqrt{7})}\,x\,\sqrt{3 - 2\,x + x^2}\Big)\Big]\Big) \Big/$
$\Big(32\,282\,885\,600\,000\,000\,000\,000\,000\,\sqrt{70\,(-5 + i\,\sqrt{7})}\Big) + \Big(\big(-232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\big)$

$$\text{Log}\left[\left(1+x+2x^2\right)\left(-163\,i+15\sqrt{7}+122\,i\,x-10\sqrt{7}\,x-41\,i\,x^2+5\sqrt{7}\,x^2-13\,i\sqrt{10\left(5+i\sqrt{7}\right)}\sqrt{3-2x+x^2}+5\,i\sqrt{10\left(5+i\sqrt{7}\right)}\,x\sqrt{3-2x+x^2}\right)\right]\Big/\left(32\,282\,885\,600\,000\,000\,000\,000\,000\sqrt{70\left(5+i\sqrt{7}\right)}\right)$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$-\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\,\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,(-a+x)}{\sqrt{(-a+x)\left(1+x^2\right)}}\right]$$

Result (type 4, 213 leaves):

$$\left(2\sqrt{\frac{a-x}{i+a}}\left(-\left(-i-a+\sqrt{1+a^2}\right)\sqrt{1+i\,x}\,(i+x)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-i\,x}}{\sqrt{2}}\right],\frac{2\,i}{i+a}\right]+2\,i\sqrt{1+a^2}\sqrt{1-i\,x}\sqrt{1+x^2}\,\text{EllipticPi}\left[\frac{2\,i}{i+a-\sqrt{1+a^2}},\text{ArcSin}\left[\frac{\sqrt{1-i\,x}}{\sqrt{2}}\right],\frac{2\,i}{i+a}\right]\right)\right)\Big/\left(\left(i+a-\sqrt{1+a^2}\right)\sqrt{1-i\,x}\sqrt{(-a+x)\left(1+x^2\right)}\right)$$

Problem 53: Result unnecessarily involves higher level functions.

$$\int\frac{a+b\,x}{\left(1-x^2\right)^{1/3}\left(3+x^2\right)}\,dx$$

Optimal (type 3, 198 leaves, 7 steps):

$$\frac{a\,\text{ArcTan}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\,b\,\text{ArcTan}\left[\frac{1+\left(2-2x^2\right)^{1/3}}{\sqrt{3}}\right]}{2\times 2^{2/3}}+\frac{a\,\text{ArcTan}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1-x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{a\,\text{ArcTanh}[x]}{6\times 2^{2/3}}+\frac{a\,\text{ArcTanh}\left[\frac{x}{1+2^{1/3}\left(1-x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}-\frac{b\,\text{Log}\left[3+x^2\right]}{4\times 2^{2/3}}+\frac{3\,b\,\text{Log}\left[2^{2/3}-\left(1-x^2\right)^{1/3}\right]}{4\times 2^{2/3}}$$

Result (type 6, 205 leaves):

$$\frac{1}{\left(1-x^2\right)^{1/3}\left(3+x^2\right)}3\,x\left(\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]\right)\Big/\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]+2\,x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right]\right)\right)+\left(b\,x\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^2,-\frac{x^2}{3}\right]\right)\Big/\left(6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^2,-\frac{x^2}{3}\right]+x^2\left(-\text{AppellF1}\left[2,\frac{1}{3},2,3,x^2,-\frac{x^2}{3}\right]+\text{AppellF1}\left[2,\frac{4}{3},1,3,x^2,-\frac{x^2}{3}\right]\right)\right)\right)$$

Problem 54: Result unnecessarily involves higher level functions.

$$\int\frac{a+b\,x}{\left(3-x^2\right)\left(1+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 198 leaves, 7 steps):

$$-\frac{a\,\text{ArcTan}[x]}{6\times 2^{2/3}}+\frac{a\,\text{ArcTan}\left[\frac{x}{1+2^{1/3}\left(1+x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}-\frac{\sqrt{3}\,b\,\text{ArcTan}\left[\frac{1+2^{1/3}\left(1+x^2\right)^{1/3}}{\sqrt{3}}\right]}{2\times 2^{2/3}}-\frac{a\,\text{ArcTanh}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{a\,\text{ArcTanh}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1+x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{b\,\text{Log}\left[3-x^2\right]}{4\times 2^{2/3}}-\frac{3\,b\,\text{Log}\left[2^{2/3}-\left(1+x^2\right)^{1/3}\right]}{4\times 2^{2/3}}$$

Result (type 6, 220 leaves):

$$\frac{1}{\left(-3+x^2\right)\left(1+x^2\right)^{1/3}}3\,x\left(-\left(\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]\right)\Big/\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-x^2,\frac{x^2}{3}\right]\right)\right)\right)-\left(b\,x\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-x^2,\frac{x^2}{3}\right]\right)\Big/\left(6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-x^2,\frac{x^2}{3}\right]+x^2\left(\text{AppellF1}\left[2,\frac{1}{3},2,3,-x^2,\frac{x^2}{3}\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,-x^2,\frac{x^2}{3}\right]\right)\right)\right)$$

Problem 55: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x\left(4-6\,x+3\,x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 97 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\left(4-6x+3x^2\right)^{1/3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}[x]}{2\times 2^{2/3}}+\frac{\text{Log}\left[6-3x-3\times 2^{1/3}\left(4-6x+3x^2\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 6, 273 leaves) :

$$-\left(\left(15x\left(-3-i\sqrt{3}+3x\right)\left(-3+i\sqrt{3}+3x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right]\right)\Big/\left(2\left(4-6x+3x^2\right)^{4/3}\left(15x\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right]+\left(3+i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right]+\left(3-i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right]\right)\right)\right)$$

Problem 56: Result unnecessarily involves higher level functions.

$$\int x\left(1-x^3\right)^{1/3}dx$$

Optimal (type 3, 73 leaves, 2 steps) :

$$\frac{1}{3}x^2\left(1-x^3\right)^{1/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{6}\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]$$

Result (type 5, 34 leaves) :

$$\frac{1}{6}x^2\left(2\left(1-x^3\right)^{1/3}+\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},x^3\right]\right)$$

Problem 57: Result unnecessarily involves higher level functions.

$$\int\frac{\left(1-x^3\right)^{1/3}}{x}dx$$

Optimal (type 3, 67 leaves, 6 steps) :

$$\left(1-x^3\right)^{1/3}-\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]$$

Result (type 5, 48 leaves) :

$$\frac{2-2x^3-\left(1-\frac{1}{x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{x^3}\right]}{2\left(1-x^3\right)^{2/3}}$$

Problem 58: Unable to integrate problem.

$$\int\frac{\left(1-x^3\right)^{1/3}}{1+x}\,dx$$

Optimal (type 3, 482 leaves, 25 steps):

$$\left(1-x^3\right)^{1/3}+\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-$$
$$\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{3}\times 2^{1/3}\,\text{Log}\left[1+x^3\right]+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{1}{3}\times 2^{1/3}\,\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]-$$
$$\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2^{2/3}}-\frac{1}{2}\,\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2^{2/3}}$$

Result (type 8, 19 leaves):

$$\int\frac{\left(1-x^3\right)^{1/3}}{1+x}\,dx$$

Problem 59: Unable to integrate problem.

$$\int\frac{\left(1-x^3\right)^{1/3}}{1-x+x^2}\,dx$$

Optimal (type 3, 280 leaves, 19 steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,2^{1/3}(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[-3\,(-1+x)\,\left(1-x+x^2\right)\right]}{2\times 2^{2/3}}+$$
$$\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[-2^{1/3}(-1+x)+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+\frac{1}{2}\,\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}x+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 24 leaves):

$$\int \frac{\left(1-x^3\right)^{1/3}}{1-x+x^2}\,dx$$

Problem 60: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{1/3}}{2+x}\,dx$$

Optimal (type 6, 232 leaves, 12 steps):

$$\left(1-x^3\right)^{1/3}+\frac{1}{2}\,x\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right]-\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+3^{1/6}\,\text{ArcTan}\left[\frac{1-\frac{3^{2/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]-3^{1/6}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\left(1-x^3\right)^{1/3}}{3\times 3^{1/6}}\right]-\frac{\text{Log}\left[8+x^3\right]}{3^{1/3}}+\frac{1}{2}\times 3^{2/3}\,\text{Log}\left[3^{2/3}-\left(1-x^3\right)^{1/3}\right]-\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{1}{2}\times 3^{2/3}\,\text{Log}\left[-\frac{1}{2}\times 3^{2/3}x-\left(1-x^3\right)^{1/3}\right]$$

Result (type 8, 19 leaves):

$$\int \frac{\left(1-x^3\right)^{1/3}}{2+x}\,dx$$

Problem 61: Unable to integrate problem.

$$\int \frac{2+x}{\left(1+x+x^2\right)\left(2+x^3\right)^{1/3}}\,dx$$

Optimal (type 6, 168 leaves, 9 steps):

$$-\frac{x^2\,\text{AppellF1}\left[\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right]}{2\times 2^{1/3}}+\frac{2\,\text{ArcTan}\left[\frac{1+\frac{2\,3^{1/3}x}{\left(2+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}+\frac{\text{ArcTan}\left[\frac{3^{1/3}+2\left(2+x^3\right)^{1/3}}{3^{5/6}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-x^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3}-\left(2+x^3\right)^{1/3}\right]}{2\times 3^{1/3}}-\frac{\text{Log}\left[3^{1/3}x-\left(2+x^3\right)^{1/3}\right]}{3^{1/3}}$$

Result (type 8, 23 leaves):

$$\int \frac{2+x}{\left(1+x+x^2\right)\left(2+x^3\right)^{1/3}}\,dx$$

Problem 63: Result is not expressed in closed-form.

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx$$

Optimal (type 3, 59 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{7-40x}{5\sqrt{11}}\right]}{2\sqrt{11}}+\frac{\text{ArcTan}\left[\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right]}{2\sqrt{11}}$$

Result (type 7, 86 leaves):

$$\frac{1}{8}\text{RootSum}\left[9+24\,\#1-12\,\#1^2+80\,\#1^3+320\,\#1^4\,\&,\ \frac{3\,\text{Log}[x-\#1]+12\,\text{Log}[x-\#1]\,\#1+20\,\text{Log}[x-\#1]\,\#1^2}{3-3\,\#1+30\,\#1^2+160\,\#1^3}\,\&\right]$$

Problem 64: Result is not expressed in closed-form.

$$\int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$$

Optimal (type 3, 78 leaves, 2 steps):

$$2\sqrt{11}\,\text{ArcTan}\left[\frac{7-40x}{5\sqrt{11}}\right]-2\sqrt{11}\,\text{ArcTan}\left[\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right]+2\,\text{Log}\left[9+24x-12x^2+80x^3+320x^4\right]$$

Result (type 7, 99 leaves):

$$\frac{1}{2}\text{RootSum}\left[9+24\,\#1-12\,\#1^2+80\,\#1^3+320\,\#1^4\,\&,\ \frac{-21\,\text{Log}[x-\#1]-144\,\text{Log}[x-\#1]\,\#1-100\,\text{Log}[x-\#1]\,\#1^2+640\,\text{Log}[x-\#1]\,\#1^3}{3-3\,\#1+30\,\#1^2+160\,\#1^3}\,\&\right]$$

Problem 65: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx$$

Optimal (type 3, 49 leaves, 1 step):

$$\frac{1}{2}\text{ArcTan}\left[\frac{x\left(1+x^2\right)}{\sqrt{1-x^4}}\right]+\frac{1}{2}\text{ArcTanh}\left[\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right]$$

Result (type 6, 110 leaves):

$$-\left(\left(5x\sqrt{1-x^4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},x^4,-x^4\right]\right)\Big/\left(\left(1+x^4\right)\left(-5\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},x^4,-x^4\right]+2x^4\left(2\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},x^4,-x^4\right]+\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},x^4,-x^4\right]\right)\right)\right)\right)$$

Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\, dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\, x}{\sqrt{1+x^4}}\right]}{2\sqrt{2}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\, x}{\sqrt{1+x^4}}\right]}{2\sqrt{2}}$$

Result (type 6, 108 leaves):

$$-\left(\left(5\, x\, \sqrt{1+x^4}\ \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-x^4,x^4\right]\right)\Big/\left(\left(-1+x^4\right)\left(5\, \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-x^4,x^4\right]+2\, x^4\left(2\, \text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-x^4,x^4\right]+\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-x^4,x^4\right]\right)\right)\right)\right)$$

Problem 67: Unable to integrate problem.

$$\int \frac{\sqrt{1+p\, x^2+x^4}}{1-x^4}\, dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$\frac{1}{4}\sqrt{2-p}\ \text{ArcTan}\left[\frac{\sqrt{2-p}\ x}{\sqrt{1+p\, x^2+x^4}}\right]+\frac{1}{4}\sqrt{2+p}\ \text{ArcTanh}\left[\frac{\sqrt{2+p}\ x}{\sqrt{1+p\, x^2+x^4}}\right]$$

Result (type 8, 26 leaves):

$$\int \frac{\sqrt{1+p\, x^2+x^4}}{1-x^4}\, dx$$

Problem 68: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{1+p\, x^2-x^4}}{1+x^4}\, dx$$

Optimal (type 3, 171 leaves, 1 step):

$$-\frac{\sqrt{p+\sqrt{4+p^2}}\ \text{ArcTan}\left[\frac{\sqrt{p+\sqrt{4+p^2}}\ x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\ \sqrt{1+p x^2-x^4}}\right]}{2\sqrt{2}}+\frac{\sqrt{-p+\sqrt{4+p^2}}\ \text{ArcTanh}\left[\frac{\sqrt{-p+\sqrt{4+p^2}}\ x\left(p+\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\ \sqrt{1+p x^2-x^4}}\right]}{2\sqrt{2}}$$

Result (type 4, 322 leaves):

$$\left(\sqrt{2+\frac{4x^2}{-p+\sqrt{4+p^2}}}\ \sqrt{1-\frac{2x^2}{p+\sqrt{4+p^2}}}\ \left(2\,i\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\ \sqrt{\frac{1}{-p+\sqrt{4+p^2}}}\ x\right],\frac{p-\sqrt{4+p^2}}{p+\sqrt{4+p^2}}\right]-\right.\right.$$
$$(2\,i+p)\ \text{EllipticPi}\left[\frac{1}{2}\,i\left(p-\sqrt{4+p^2}\right),\ i\,\text{ArcSinh}\left[\sqrt{2}\ \sqrt{\frac{1}{-p+\sqrt{4+p^2}}}\ x\right],\frac{p-\sqrt{4+p^2}}{p+\sqrt{4+p^2}}\right]+$$
$$\left.\left.(-2\,i+p)\ \text{EllipticPi}\left[\frac{1}{2}\,i\left(-p+\sqrt{4+p^2}\right),\ i\,\text{ArcSinh}\left[\sqrt{2}\ \sqrt{\frac{1}{-p+\sqrt{4+p^2}}}\ x\right],\frac{p-\sqrt{4+p^2}}{p+\sqrt{4+p^2}}\right]\right)\right)\Bigg/\left(4\sqrt{\frac{1}{-p+\sqrt{4+p^2}}}\ \sqrt{1+p x^2-x^4}\right)$$

Problem 69: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{a+b x}{\left(2-x^2\right)\left(-1+x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 80 leaves, 7 steps):

$$\frac{a\,\text{ArcTan}\left[\frac{x}{\sqrt{2}\left(-1+x^2\right)^{1/4}}\right]}{2\sqrt{2}}-b\,\text{ArcTan}\left[\left(-1+x^2\right)^{1/4}\right]+\frac{a\,\text{ArcTanh}\left[\frac{x}{\sqrt{2}\left(-1+x^2\right)^{1/4}}\right]}{2\sqrt{2}}+b\,\text{ArcTanh}\left[\left(-1+x^2\right)^{1/4}\right]$$

Result (type 6, 203 leaves):

$$\frac{1}{\left(-2+x^2\right)\left(-1+x^2\right)^{1/4}}2x\left(-\left(\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right]\right)\Big/\right.\right.$$
$$\left.\left(6\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},x^2,\frac{x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},x^2,\frac{x^2}{2}\right]\right)\right)\right)-$$
$$\left.\frac{2\,b\,x\,\text{AppellF1}\left[1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right]}{8\,\text{AppellF1}\left[1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right]+x^2\left(2\,\text{AppellF1}\left[2,\frac{1}{4},2,3,x^2,\frac{x^2}{2}\right]+\text{AppellF1}\left[2,\frac{5}{4},1,3,x^2,\frac{x^2}{2}\right]\right)}\right)$$

Problem 70: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a+b x}{\left(-1-x^2\right)^{1/4} \left(2+x^2\right)} \, dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{a \operatorname{ArcTan}\left[\frac{x}{\sqrt{2} \left(-1-x^2\right)^{1/4}}\right]}{2 \sqrt{2}}+b \operatorname{ArcTan}\left[\left(-1-x^2\right)^{1/4}\right]+\frac{a \operatorname{ArcTanh}\left[\frac{x}{\sqrt{2} \left(-1-x^2\right)^{1/4}}\right]}{2 \sqrt{2}}-b \operatorname{ArcTanh}\left[\left(-1-x^2\right)^{1/4}\right]$$

Result (type 6, 221 leaves):

$$\frac{1}{\left(-1-x^2\right)^{1/4} \left(2+x^2\right)} 2 x \left(-\left(\left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}, -x^2, -\frac{x^2}{2}\right]\right) \Big/ \left(-6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}, -x^2, -\frac{x^2}{2}\right]+ x^2 \left(2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{4}, 2, \frac{5}{2}, -x^2, -\frac{x^2}{2}\right]+\operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{4}, 1, \frac{5}{2}, -x^2, -\frac{x^2}{2}\right]\right)\right)\right)-\left(2 b x \operatorname{AppellF1}\left[1, \frac{1}{4}, 1, 2, -x^2, -\frac{x^2}{2}\right]\right) \Big/ \left(-8 \operatorname{AppellF1}\left[1, \frac{1}{4}, 1, 2, -x^2, -\frac{x^2}{2}\right]+x^2 \left(2 \operatorname{AppellF1}\left[2, \frac{1}{4}, 2, 3, -x^2, -\frac{x^2}{2}\right]+\operatorname{AppellF1}\left[2, \frac{5}{4}, 1, 3, -x^2, -\frac{x^2}{2}\right]\right)\right)\right)$$

Problem 71: Result unnecessarily involves higher level functions.

$$\int \frac{a+b x}{\left(1-x^2\right)^{1/4} \left(2-x^2\right)} \, dx$$

Optimal (type 3, 149 leaves, 3 steps):

$$\frac{b \operatorname{ArcTan}\left[\frac{1-\sqrt{1-x^2}}{\sqrt{2} \left(1-x^2\right)^{1/4}}\right]}{\sqrt{2}}+\frac{1}{2} a \operatorname{ArcTan}\left[\frac{1-\sqrt{1-x^2}}{x \left(1-x^2\right)^{1/4}}\right]+\frac{b \operatorname{ArcTanh}\left[\frac{1+\sqrt{1-x^2}}{\sqrt{2} \left(1-x^2\right)^{1/4}}\right]}{\sqrt{2}}+\frac{1}{2} a \operatorname{ArcTanh}\left[\frac{1+\sqrt{1-x^2}}{x \left(1-x^2\right)^{1/4}}\right]$$

Result (type 6, 205 leaves):

$$\frac{1}{\left(1-x^2\right)^{1/4} \left(-2+x^2\right)} 2 x \left(-\left(\left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}, x^2, \frac{x^2}{2}\right]\right) \Big/ \left(6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}, x^2, \frac{x^2}{2}\right]+x^2 \left(2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{4}, 2, \frac{5}{2}, x^2, \frac{x^2}{2}\right]+\operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{4}, 1, \frac{5}{2}, x^2, \frac{x^2}{2}\right]\right)\right)\right)- \frac{2 b x \operatorname{AppellF1}\left[1, \frac{1}{4}, 1, 2, x^2, \frac{x^2}{2}\right]}{8 \operatorname{AppellF1}\left[1, \frac{1}{4}, 1, 2, x^2, \frac{x^2}{2}\right]+x^2 \left(2 \operatorname{AppellF1}\left[2, \frac{1}{4}, 2, 3, x^2, \frac{x^2}{2}\right]+\operatorname{AppellF1}\left[2, \frac{5}{4}, 1, 3, x^2, \frac{x^2}{2}\right]\right)}\right)$$

Problem 72: Result unnecessarily involves higher level functions.

$$\int \frac{a+b x}{\left(1+x^2\right)^{1/4} \left(2+x^2\right)} \, dx$$

Optimal (type 3, 135 leaves, 3 steps):

$$-\frac{b \operatorname{ArcTan}\left[\frac{1-\sqrt{1+x^2}}{\sqrt{2} \left(1+x^2\right)^{1/4}}\right]}{\sqrt{2}}-\frac{1}{2} a \operatorname{ArcTan}\left[\frac{1+\sqrt{1+x^2}}{x \left(1+x^2\right)^{1/4}}\right]-\frac{1}{2} a \operatorname{ArcTanh}\left[\frac{1-\sqrt{1+x^2}}{x \left(1+x^2\right)^{1/4}}\right]-\frac{b \operatorname{ArcTanh}\left[\frac{1+\sqrt{1+x^2}}{\sqrt{2} \left(1+x^2\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 6, 219 leaves):

$$\frac{1}{\left(1+x^2\right)^{1/4} \left(2+x^2\right)} 2 x \left(-\left(\left(3 a \operatorname{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right]\right) \middle/ \left(-6 \operatorname{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right]+x^2 \left(2 \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-x^2,-\frac{x^2}{2}\right]+\operatorname{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-x^2,-\frac{x^2}{2}\right]\right)\right)\right)-\left(2 b x \operatorname{AppellF1}\left[1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right]\right) \middle/ \left(-8 \operatorname{AppellF1}\left[1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right]+x^2 \left(2 \operatorname{AppellF1}\left[2,\frac{1}{4},2,3,-x^2,-\frac{x^2}{2}\right]+\operatorname{AppellF1}\left[2,\frac{5}{4},1,3,-x^2,-\frac{x^2}{2}\right]\right)\right)\right)$$

Problem 73: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{1-x^3} \left(4-x^3\right)} \, dx$$

Optimal (type 3, 127 leaves, 1 step):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{3} \left(1-2^{1/3} x\right)}{\sqrt{1-x^3}}\right]}{3 \times 2^{2/3} \sqrt{3}}+\frac{\operatorname{ArcTan}\left[\frac{\sqrt{1-x^3}}{\sqrt{3}}\right]}{3 \times 2^{2/3} \sqrt{3}}-\frac{\operatorname{ArcTanh}\left[\frac{1+2^{1/3} x}{\sqrt{1-x^3}}\right]}{3 \times 2^{2/3}}+\frac{\operatorname{ArcTanh}\left[\sqrt{1-x^3}\right]}{9 \times 2^{2/3}}$$

Result (type 6, 120 leaves):

$$-\left(\left(10 x^2 \operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right]\right) \middle/ \left(\sqrt{1-x^3} \left(-4+x^3\right) \left(20 \operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right]+3 x^3 \left(\operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,\frac{x^3}{4}\right]+2 \operatorname{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,\frac{x^3}{4}\right]\right)\right)\right)\right)$$

Problem 74: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(4-d x^3\right) \sqrt{-1+d x^3}} \, dx$$

Optimal (type 3, 157 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+2^{1/3} d^{1/3} x}{\sqrt{-1+d x^3}}\right]}{3\times 2^{2/3} d^{2/3}}-\frac{\text{ArcTan}\left[\sqrt{-1+d x^3}\right]}{9\times 2^{2/3} d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\left(1-2^{1/3} d^{1/3} x\right)}{\sqrt{-1+d x^3}}\right]}{3\times 2^{2/3}\sqrt{3}\, d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{-1+d x^3}}{\sqrt{3}}\right]}{3\times 2^{2/3}\sqrt{3}\, d^{2/3}}$$

Result (type 6, 135 leaves):

$$-\left(\left(10 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},d x^3,\frac{d x^3}{4}\right]\right)\Big/\left(\left(-4+d x^3\right)\sqrt{-1+d x^3}\right.\right.$$
$$\left.\left.\left(20 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},d x^3,\frac{d x^3}{4}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},d x^3,\frac{d x^3}{4}\right]+2 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},d x^3,\frac{d x^3}{4}\right]\right)\right)\right)\right)$$

Problem 75: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{-1+x^3}\left(8+x^3\right)} \, dx$$

Optimal (type 3, 74 leaves, 8 steps):

$$\frac{1}{18}\text{ArcTan}\left[\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right]+\frac{1}{18}\text{ArcTan}\left[\frac{1}{3}\sqrt{-1+x^3}\right]-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,(1-x)}{\sqrt{-1+x^3}}\right]}{6\sqrt{3}}$$

Result (type 6, 118 leaves):

$$-\left(\left(20 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{8}\right]\right)\Big/\right.$$
$$\left.\left(\sqrt{-1+x^3}\left(8+x^3\right)\left(-40 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{8}\right]+3 x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,-\frac{x^3}{8}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,-\frac{x^3}{8}\right]\right)\right)\right)\right)$$

Problem 76: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(8-d x^3\right)\sqrt{1+d x^3}} \, dx$$

Optimal (type 3, 103 leaves, 8 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,\left(1+d^{1/3}\,x\right)}{\sqrt{1+d\,x^3}}\right]}{6\,\sqrt{3}\,d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\left(1+d^{1/3}\,x\right)^2}{3\,\sqrt{1+d\,x^3}}\right]}{18\,d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{1}{3}\,\sqrt{1+d\,x^3}\right]}{18\,d^{2/3}}$$

Result (type 6, 139 leaves):

$$-\left(\left(20\,x^2\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{2},\,1,\,\frac{5}{3},\,-d\,x^3,\,\frac{d\,x^3}{8}\right]\right)\Big/\left(\left(-8+d\,x^3\right)\,\sqrt{1+d\,x^3}\right.\right.$$
$$\left.\left.\left(40\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{2},\,1,\,\frac{5}{3},\,-d\,x^3,\,\frac{d\,x^3}{8}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,2,\,\frac{8}{3},\,-d\,x^3,\,\frac{d\,x^3}{8}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\,\frac{3}{2},\,1,\,\frac{8}{3},\,-d\,x^3,\,\frac{d\,x^3}{8}\right]\right)\right)\right)\right)$$

Problem 77: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(1-3\,x^2\right)^{1/3}\,\left(3-x^2\right)}\,dx$$

Optimal (type 3, 81 leaves, 1 step):

$$\frac{1}{4}\,\text{ArcTan}\left[\frac{1-\left(1-3\,x^2\right)^{1/3}}{x}\right]+\frac{\text{ArcTanh}\left[\frac{x}{\sqrt{3}}\right]}{4\,\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{\left(1-\left(1-3\,x^2\right)^{1/3}\right)^2}{3\,\sqrt{3}\,x}\right]}{4\,\sqrt{3}}$$

Result (type 6, 126 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\,\frac{1}{3},\,1,\,\frac{3}{2},\,3\,x^2,\,\frac{x^2}{3}\right]\right)\Big/\right.$$
$$\left.\left(\left(1-3\,x^2\right)^{1/3}\,\left(-3+x^2\right)\,\left(9\,\text{AppellF1}\left[\frac{1}{2},\,\frac{1}{3},\,1,\,\frac{3}{2},\,3\,x^2,\,\frac{x^2}{3}\right]+2\,x^2\,\left(\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{3},\,2,\,\frac{5}{2},\,3\,x^2,\,\frac{x^2}{3}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\,\frac{4}{3},\,1,\,\frac{5}{2},\,3\,x^2,\,\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 78: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(3+x^2\right)\,\left(1+3\,x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 81 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{x}{\sqrt{3}}\right]}{4\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\left(1-\left(1+3\,x^2\right)^{1/3}\right)^2}{3\,\sqrt{3}\,x}\right]}{4\,\sqrt{3}}-\frac{1}{4}\,\text{ArcTanh}\left[\frac{1-\left(1+3\,x^2\right)^{1/3}}{x}\right]$$

Result (type 6, 126 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3\,x^2,-\frac{x^2}{3}\right]\right)\Big/\left(\left(3+x^2\right)\,\left(1+3\,x^2\right)^{1/3}\left(-9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3\,x^2,-\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-3\,x^2,-\frac{x^2}{3}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-3\,x^2,-\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 79: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(1-x^2\right)^{1/3}\left(3+x^2\right)}\,dx$$

Optimal (type 3, 113 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1-x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{\text{ArcTanh}[x]}{6\times 2^{2/3}}+\frac{\text{ArcTanh}\left[\frac{x}{1+2^{1/3}\left(1-x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}$$

Result (type 6, 118 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]\right)\Big/\left(\left(1-x^2\right)^{1/3}\left(3+x^2\right)\left(-9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 80: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(3-x^2\right)\left(1+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 109 leaves, 1 step):

$$-\frac{\text{ArcTan}[x]}{6\times 2^{2/3}}+\frac{\text{ArcTan}\left[\frac{x}{1+2^{1/3}\left(1+x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1+x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}$$

Result (type 6, 124 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]\right)\Big/\left(\left(-3+x^2\right)\left(1+x^2\right)^{1/3}\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-x^2,\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 81: Result unnecessarily involves higher level functions.

$$\int \frac{a+x}{(-a+x)\sqrt{a^2 x-(1+a^2) x^2+x^3}}\,dx$$

Optimal (type 3, 87 leaves, 4 steps):

$$-\frac{2\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\;\text{ArcTan}\left[\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)x+x^2}}\right]}{(1-a)\sqrt{a^2 x-(1+a^2)x^2+x^3}}$$

Result (type 4, 159 leaves):

$$-\left(\left(2\,i\,(a^2-x)^{3/2}\sqrt{\frac{-1+x}{-a^2+x}}\sqrt{\frac{x}{-a^2+x}}\left((1+a)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right],1-\frac{1}{a^2}\right]-2\,\text{EllipticPi}\left[\frac{-1+a}{a},i\,\text{ArcSinh}\left[\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right],1-\frac{1}{a^2}\right]\right)\right)\Big/\left((-1+a)\sqrt{-a^2}\sqrt{(-1+x)\,x\,(-a^2+x)}\right)\right)$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)\,a\,x+(-1-2a+a^2)\,x^2+x^3}}\,dx$$

Optimal (type 1, 1 leaves, ? steps):

$$0$$

Result (type 4, 100 leaves):

$$-\left(\left(2\,i\sqrt{1+\frac{1}{-1+x}}\sqrt{1+\frac{(-1+a)^2}{-1+x}}\,(-1+x)^{3/2}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right],(-1+a)^2\right]-2\,\text{EllipticPi}\left[1-a,i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right],(-1+a)^2\right]\right)\right)\Big/\left(\sqrt{(-1+x)\,x\,(-2a+a^2+x)}\right)\right)$$

Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}}\,dx$$

Optimal (type 3, 46 leaves, ? steps):

$$\text{Log}\left[\frac{-a^2+2ax+x^2-2\left(x+\sqrt{(1-x)x(a^2+x-2ax)}\right)}{(a-x)^2}\right]$$

Result (type 4, 133 leaves):

$$\left(2\,i\,(-1+x)^{3/2}\sqrt{\frac{x}{-1+x}}\sqrt{-\frac{a^2+x-2ax}{(-1+2a)(-1+x)}}\right.$$
$$\left.\left(-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right],-\frac{(-1+a)^2}{-1+2a}\right]+2a\,\text{EllipticPi}\left[1-a,\,i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right],-\frac{(-1+a)^2}{-1+2a}\right]\right)\right)\Big/$$
$$\left(\sqrt{-(-1+x)x(a^2+x-2ax)}\right)$$

Problem 84: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-2^{1/3}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx$$

Optimal (type 3, 32 leaves, 2 steps):

$$\frac{2\,\text{ArcTan}\left[\frac{\sqrt{3}\,(1+2^{1/3}x)}{\sqrt{1+x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 323 leaves):

$$-\left(\left(2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left(6i+3i\,2^{1/3}-2\sqrt{3}+2^{1/3}\sqrt{3}+\left(-3i\,2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3}\right)x\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\,3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]-6i\sqrt{3}\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i\,2^{2/3}+\sqrt{3}},\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\,3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right)\right)\Big/\left(\left(1+2\times2^{2/3}-i\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}\right)\right)$$

Problem 85: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$-\frac{2}{3}\text{ArcTanh}\left[\frac{(1+x)^2}{3\sqrt{1+x^3}}\right]$$

Result (type 4, 262 leaves):

$$\left(2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left(1+i\sqrt{3}+x-i\sqrt{3}x\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\,3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]-2\sqrt{3}\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\text{EllipticPi}\left[\frac{2\sqrt{3}}{-3i+\sqrt{3}},\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\,3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right)\right)\Big/\left(\left(-3i+\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}\right)$$

Problem 86: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$$

Optimal (type 3, 218 leaves, 1 step):

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right]}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}\,3^{3/4}}\right]}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right]}{3\sqrt{2}\,3^{1/4}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right]}{6\sqrt{2}\,3^{1/4}}$$

Result (type 6, 206 leaves):

$$-\left(\left(10\left(26+15\sqrt{3}\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right]\right)\Big/\right.$$
$$\left(\left(5+3\sqrt{3}\right)\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)\left(-10\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right]+\right.\right.$$
$$\left.\left.\left.3x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right]\right)\right)\right)\right)$$

Problem 87: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx$$

Optimal (type 3, 210 leaves, 1 step):

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right]}{3\sqrt{2}\,3^{1/4}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right]}{6\sqrt{2}\,3^{1/4}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right]}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}\,3^{3/4}}\right]}{3\sqrt{2}\,3^{3/4}}$$

Result (type 6, 207 leaves):

$$\left(10\left(26-15\sqrt{3}\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]\right)\Big/$$
$$\left(\left(-5+3\sqrt{3}\right)\left(-10+6\sqrt{3}-x^3\right)\sqrt{1+x^3}\left(\left(50-30\sqrt{3}\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]-\right.\right.$$
$$\left.\left.3x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]\right)\right)\right)$$

Problem 88: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$$

Optimal (type 3, 222 leaves, 1 step):

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right]}{6\sqrt{2}\,3^{1/4}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right]}{3\sqrt{2}\,3^{1/4}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right]}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}\,3^{3/4}}\right]}{3\sqrt{2}\,3^{3/4}}$$

Result (type 6, 196 leaves):

$$-\left(\left(10\left(26+15\sqrt{3}\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{10+6\sqrt{3}}\right]\right)\Big/\left(\left(5+3\sqrt{3}\right)\left(10+6\sqrt{3}-x^3\right)\sqrt{-1+x^3}\left(10\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{10+6\sqrt{3}}\right]+3x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,\frac{x^3}{10+6\sqrt{3}}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,\frac{x^3}{10+6\sqrt{3}}\right]\right)\right)\right)\right)$$

Problem 89: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$$

Optimal (type 3, 214 leaves, 1 step):

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right]}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}\,3^{3/4}}\right]}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right]}{6\sqrt{2}\,3^{1/4}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right]}{3\sqrt{2}\,3^{1/4}}$$

Result (type 6, 198 leaves):

$$\left(10\left(26-15\sqrt{3}\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]\right)\Big/\left(\left(-5+3\sqrt{3}\right)\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)\left(10\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]-3x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,-\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,-\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]\right)\right)\right)$$

Problem 90: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx$$

Optimal (type 3, 65 leaves, 2 steps):

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\ \text{ArcTanh}\left[\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\ \sqrt{-4+4\sqrt{3}\ x^2+x^4}}\right]$$

Result (type 4, 685 leaves) :

$$\left(\left(-1+\sqrt{3}+x\right)^2 \sqrt{2\left(1+\sqrt{3}\right)-2\left(2+\sqrt{3}\right)x+\left(-1+\sqrt{3}\right)x^2-x^3}\ \sqrt{\frac{1+\sqrt{3}-\frac{4}{-1+\sqrt{3}+x}}{3+\sqrt{3}+\mathbb{i}\sqrt{2\left(2+\sqrt{3}\right)}}}\right.$$

$$\left(\left(\mathbb{i}\left(-1+\sqrt{3}+\mathbb{i}\sqrt{2\left(2+\sqrt{3}\right)}\right)+\frac{2\left(2\,\mathbb{i}\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right)}\right)}{-1+\sqrt{3}+x}\right)\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}+\mathbb{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-\mathbb{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\frac{2\,\mathbb{i}\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+\mathbb{i}\sqrt{2\left(2+\sqrt{3}\right)}}\right]+$$

$$2\sqrt{6}\sqrt{\frac{4+2\sqrt{3}+x^2}{\left(-1+\sqrt{3}+x\right)^2}}\ \sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-\mathbb{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}$$

$$\left.\left.\text{EllipticPi}\left[\frac{2\sqrt{2\left(2+\sqrt{3}\right)}}{\sqrt{2\left(2+\sqrt{3}\right)}+\mathbb{i}\left(3+\sqrt{3}\right)},\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-\mathbb{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\frac{2\,\mathbb{i}\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+\mathbb{i}\sqrt{2\left(2+\sqrt{3}\right)}}\right]\right)\right)\Big/$$

$$\left(\left(\sqrt{2\left(2+\sqrt{3}\right)}+\mathbb{i}\left(3+\sqrt{3}\right)\right)\sqrt{1+\sqrt{3}-\left(2+\sqrt{3}\right)x+\frac{1}{2}\left(-1+\sqrt{3}\right)x^2-\frac{x^3}{2}}\ \sqrt{-4+4\sqrt{3}\,x^2+x^4}\ \sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-\mathbb{i}\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}\right)$$

Problem 91: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx$$

Optimal (type 3, 63 leaves, 2 steps):

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\text{ArcTan}\left[\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\right]$$

Result (type 4, 1137 leaves):

$$-\left(\left(\left(-1-\sqrt{3}+x\right)^2\sqrt{\frac{-1+\sqrt{3}+\frac{4}{-1-\sqrt{3}+x}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\sqrt{-24+16\sqrt{3}+\left(20-8\sqrt{3}\right)\left(1-\sqrt{3}+x\right)+\left(-2+4\sqrt{3}\right)\left(1-\sqrt{3}+x\right)^2+\left(1-\sqrt{3}+x\right)^3}\right.\right.$$

$$\left(\left(i\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8\,i}{-1-\sqrt{3}+x}}+i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8\,i}{-1-\sqrt{3}+x}}+\right.\right.$$

$$\sqrt{-2\,i+2\,i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16\,i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}+\frac{1}{-1-\sqrt{3}+x}$$

$$2\left(2\,i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8\,i}{-1-\sqrt{3}+x}}+\sqrt{6}\sqrt{-i+i\sqrt{3}-\sqrt{12-6\sqrt{3}}+2\sqrt{4-2\sqrt{3}}-\frac{8\,i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}+\right.$$

$$\left.\left.\sqrt{-2\,i+2\,i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16\,i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}\right)\right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8\,i}{-1-\sqrt{3}+x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]+$$

$$2\sqrt{6}\,\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8\,i}{-1-\sqrt{3}+x}}\,\sqrt{1+\frac{8}{\left(-1-\sqrt{3}+x\right)^{2}}+\frac{2\left(1+\sqrt{3}\right)}{-1-\sqrt{3}+x}}$$

$$\text{EllipticPi}\left[\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)},\ \text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8\,i}{-1-\sqrt{3}+x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\ \frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\left(\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)\right)\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8\,i}{-1-\sqrt{3}+x}}$$

$$\sqrt{8\left(1+\sqrt{3}\right)+4\left(3+\sqrt{3}\right)\left(-1-\sqrt{3}+x\right)+2\left(1+\sqrt{3}\right)\left(-1-\sqrt{3}+x\right)^{2}+\frac{1}{2}\left(-1-\sqrt{3}+x\right)^{3}}$$

$$\sqrt{\Big(48-32\sqrt{3}-64\left(1-\sqrt{3}+x\right)+32\sqrt{3}\left(1-\sqrt{3}+x\right)+24\left(1-\sqrt{3}+x\right)^{2}-}$$

$$\overline{16\sqrt{3}\left(1-\sqrt{3}+x\right)^{2}-4\left(1-\sqrt{3}+x\right)^{3}+4\sqrt{3}\left(1-\sqrt{3}+x\right)^{3}+\left(1-\sqrt{3}+x\right)^{4}\Big)}\Bigg)$$

Problem 92: Unable to integrate problem.

$$\int\frac{-1+x}{(1+x)\left(2+x^{3}\right)^{1/3}}\,dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(2+x)}{\left(2+x^{3}\right)^{1/3}}}{\sqrt{3}}\right]+\text{Log}[1+x]-\frac{3}{2}\,\text{Log}\left[2+x-\left(2+x^{3}\right)^{1/3}\right]$$

Result (type 8, 20 leaves):

$$\int\frac{-1+x}{(1+x)\left(2+x^{3}\right)^{1/3}}\,dx$$

Problem 93: Unable to integrate problem.

$$\int \frac{1}{(1+x)\,(2+x^3)^{1/3}}\,dx$$

Optimal (type 3, 108 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2x}{(2+x^3)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(2+x)}{(2+x^3)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\text{Log}[1+x]+\frac{3}{4}\text{Log}\left[2+x-(2+x^3)^{1/3}\right]-\frac{1}{4}\text{Log}\left[-x+(2+x^3)^{1/3}\right]$$

Result (type 8, 17 leaves):

$$\int \frac{1}{(1+x)\,(2+x^3)^{1/3}}\,dx$$

Problem 95: Unable to integrate problem.

$$\int \frac{1+x}{(1+x+x^2)\,(a+b\,x^3)^{1/3}}\,dx$$

Optimal (type 3, 154 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2(a+b)^{1/3}x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(a+b)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2(a+b\,x^3)^{1/3}}{(a+b)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(a+b)^{1/3}}+\frac{\text{Log}\left[(a+b)^{1/3}-(a+b\,x^3)^{1/3}\right]}{2\,(a+b)^{1/3}}-\frac{\text{Log}\left[(a+b)^{1/3}x-(a+b\,x^3)^{1/3}\right]}{2\,(a+b)^{1/3}}$$

Result (type 8, 25 leaves):

$$\int \frac{1+x}{(1+x+x^2)\,(a+b\,x^3)^{1/3}}\,dx$$

Problem 96: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(1-x^3)\,(a+b\,x^3)^{1/3}}\,dx$$

Optimal (type 3, 96 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2(a+b\,x^3)^{1/3}}{(a+b)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(a+b)^{1/3}}+\frac{\text{Log}\left[1-x^3\right]}{6\,(a+b)^{1/3}}-\frac{\text{Log}\left[(a+b)^{1/3}-(a+b\,x^3)^{1/3}\right]}{2\,(a+b)^{1/3}}$$

Result (type 3, 137 leaves):

$$-\frac{1}{6\,(a+b)^{1/3}}(-1)^{1/3}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\frac{2\,(-1)^{1/3}\,(a+b\,x^3)^{1/3}}{(a+b)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[(a+b)^{1/3}+(-1)^{1/3}\,(a+b\,x^3)^{1/3}\right]+\text{Log}\left[(a+b)^{2/3}-(-1)^{1/3}\,(a+b)^{1/3}\,(a+b\,x^3)^{1/3}+(-1)^{2/3}\,(a+b\,x^3)^{2/3}\right]\right)$$

Problem 98: Result unnecessarily involves higher level functions.

$$\int\frac{x}{(1-x^3)^{1/3}\,(1+x^3)}\,dx$$

Optimal (type 3, 233 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 115 leaves):

$$-\left(\left(5\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\left(2\,(1-x^3)^{1/3}\,(1+x^3)\left(-5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 100: Unable to integrate problem.

$$\int\frac{1+x}{(1-x+x^2)\,(1-x^3)^{1/3}}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 8, 27 leaves):

$$\int\frac{1+x}{(1-x+x^2)\,(1-x^3)^{1/3}}\,dx$$

Problem 101: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1+x)^2}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{2^{1/3}}$$

Result (type 6, 315 leaves) :

$$-\left(\left(5x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left.\left(\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)\right)-$$
$$\left(2x^3\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]\right)\Big/\left(\left(1-x^3\right)^{1/3}\left(1+x^3\right)\right.$$
$$\left.\left(-6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[2,\frac{1}{3},2,3,x^3,-x^3\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,x^3,-x^3\right]\right)\right)\right)+$$
$$\frac{2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\frac{2\cdot 2^{1/3}x}{\left(-1+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(-1+x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(-1+x^3\right)^{1/3}}\right]+2\,\text{Log}\left[1+\frac{2^{1/3}x}{\left(-1+x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}$$

Problem 102: Unable to integrate problem.

$$\int \frac{1-x}{\left(1+x+x^2\right)\left(1+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 119 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}(1+x)}{\left(1+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1+x)^2}{\left(1+x^3\right)^{2/3}}-\frac{2^{1/3}(1+x)}{\left(1+x^3\right)^{1/3}}\right]}{2\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1+x)}{\left(1+x^3\right)^{1/3}}\right]}{2^{1/3}}$$

Result (type 8, 25 leaves) :

$$\int \frac{1-x}{\left(1+x+x^2\right)\left(1+x^3\right)^{1/3}}\,dx$$

Problem 105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1-x)^2}{\left(1-x^3\right)^{4/3}}\,dx$$

Optimal (type 5, 39 leaves, 3 steps):

$$\frac{1+(1-2x)\,x}{\left(1-x^3\right)^{1/3}}+x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]$$

Result (type 6, 557 leaves):

$$\frac{1}{10\left(1-x^3\right)^{4/3}}(-1+x)^2\left(10\,(1+2x)\left(1+x+x^2\right)-\left(225\left(i+\sqrt{3}+2\,i\,x\right)\left(1+i\sqrt{3}+2x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]\right)\Big/\right.$$
$$\left(30\,i\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]+\left(-3\,i+\sqrt{3}\right)(-1+x)\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]-\right.$$
$$\left.\left(3\,i+\sqrt{3}\right)(-1+x)\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]\right)-$$
$$\left(144\left(i+\sqrt{3}+2\,i\,x\right)(-1+x)\left(1+i\sqrt{3}+2x\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{1}{3},\frac{8}{3},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]\right)\Big/$$
$$\left(48\,i\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{1}{3},\frac{8}{3},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]+\left(-3\,i+\sqrt{3}\right)(-1+x)\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},\frac{4}{3},\frac{11}{3},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]-\right.$$
$$\left.\left.\left(3\,i+\sqrt{3}\right)(-1+x)\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},\frac{1}{3},\frac{11}{3},\frac{2\,i\,(-1+x)}{-3\,i+\sqrt{3}},-\frac{2\,i\,(-1+x)}{3\,i+\sqrt{3}}\right]\right)\right)$$

Problem 106: Result unnecessarily involves higher level functions.

$$\int \left(1-x^3\right)^{2/3}\,dx$$

Optimal (type 3, 67 leaves, 2 steps):

$$\frac{1}{3}x\left(1-x^3\right)^{2/3}-\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{1}{3}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 101 leaves):

$$\frac{3\,(-1+x)\,\left(1-x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},-\frac{2}{3},\frac{8}{3},-\frac{-1+x}{1-(-1)^{2/3}},-\frac{-1+x}{1+(-1)^{1/3}}\right]}{5\,\left(1+\frac{-1+x}{1+(-1)^{1/3}}\right)^{2/3}\,\left(1+\frac{-1+x}{1-(-1)^{2/3}}\right)^{2/3}}$$

Problem 107: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1-x^3\right)^{2/3}}{x}\,dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{1}{2}\,\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]$$

Result (type 5, 48 leaves):

$$\frac{1-x^3-2\left(1-\frac{1}{x^3}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1}{x^3}\right]}{2\left(1-x^3\right)^{1/3}}$$

Problem 108: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+b\,x}\,dx$$

Optimal (type 6, 384 leaves, 13 steps):

$$\frac{\left(1-x^3\right)^{2/3}}{2\,b}-\frac{\left(a^3+b^3\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right]}{2\,a^2\,b^2}+\frac{a^2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}-\frac{\left(a^3+b^3\right)^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\left(a^3+b^3\right)^{1/3}x}{a\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}+$$

$$\frac{\left(a^3+b^3\right)^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2b\left(1-x^3\right)^{1/3}}{\left(a^3+b^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}+\frac{a\,x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]}{2\,b^2}-\frac{\left(a^3+b^3\right)^{2/3}\,\text{Log}\left[a^3+b^3x^3\right]}{3\,b^3}+$$

$$\frac{\left(a^3+b^3\right)^{2/3}\,\text{Log}\left[-\frac{\left(a^3+b^3\right)^{1/3}x}{a}-\left(1-x^3\right)^{1/3}\right]}{2\,b^3}-\frac{a^2\,\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]}{2\,b^3}+\frac{\left(a^3+b^3\right)^{2/3}\,\text{Log}\left[\left(a^3+b^3\right)^{1/3}-b\left(1-x^3\right)^{1/3}\right]}{2\,b^3}$$

Result (type 8, 21 leaves):

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+b\,x}\,dx$$

Problem 109: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\, dx$$

Optimal (type 5, 234 leaves, 13 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}+\frac{x\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}+\frac{2x^2\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{3\sqrt{3}}+$$
$$\frac{1}{3}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{3\times 2^{1/3}}$$

Result (type 8, 24 leaves):

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\, dx$$

Problem 110: Unable to integrate problem.

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\, dx$$

Optimal (type 3, 199 leaves, 14 steps):

$$\frac{\left(1-x^3\right)^{2/3}}{1-x+x^2}-\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+$$
$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}+\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 8, 29 leaves):

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\, dx$$

Problem 111: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$$

Optimal (type 5, 177 leaves, 5 steps):

$$\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{2\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]+\frac{3\,\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$$

Problem 112: Unable to integrate problem.

$$\int \frac{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 177 leaves, 6 steps):

$$\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{2\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]+\frac{3\,\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 8, 29 leaves):

$$\int \frac{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Problem 113: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 3, 132 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 111 leaves):

$$-\left(\left(4x\left(1-x^3\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-4\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},x^3,-x^3\right]+2\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 114: Result unnecessarily involves higher level functions.

$$\int\frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 250 leaves, 10 steps):

$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$

$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 6, 115 leaves):

$$-\left(\left(5x^2\left(1-x^3\right)^{2/3}\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\left(2\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},2,\frac{8}{3},x^3,-x^3\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 115: Result unnecessarily involves higher level functions.

$$\int\frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 383 leaves, ? steps):

$$-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+x^3\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+$$

$$\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]+\frac{\text{Log}\left[-2^{1/3}x-(1-x^3)^{1/3}\right]}{2^{1/3}}-\frac{1}{2}\text{Log}\left[x+(1-x^3)^{1/3}\right]+\frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 6, 209 leaves):

$$-\frac{1}{2(1+x^3)}x\,(1-x^3)^{2/3}\left(\left(8\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right]\right)\Big/\left(-4\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right]+\right.\right.$$

$$\left.x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},x^3,-x^3\right]+2\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\right)-\left(5\,x\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/$$

$$\left.\left(-5\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},2,\frac{8}{3},x^3,-x^3\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)$$

Problem 116: Result unnecessarily involves higher level functions.

$$\int\frac{(1-x^3)^{1/3}}{1+x^3}\,dx$$

Optimal (type 3, 272 leaves, 14 steps):

$$\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}-$$

$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{1}{3}\times 2^{1/3}\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}$$

Result (type 6, 109 leaves):

$$-\left(\left(4\,x\,(1-x^3)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right]\right)\Big/\right.$$

$$\left.\left((1+x^3)\left(-4\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right]+\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Test results for the 8 problems in "Wester Problems.m"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{3+3\,\mathrm{Cos}[x]+4\,\mathrm{Sin}[x]}\,dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{1}{4}\,\mathrm{Log}\left[3+4\,\mathrm{Tan}\left[\frac{x}{2}\right]\right]$$

Result (type 3, 34 leaves):

$$-\frac{1}{4}\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]\right]+\frac{1}{4}\,\mathrm{Log}\left[3\,\mathrm{Cos}\left[\frac{x}{2}\right]+4\,\mathrm{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{5+3\,\mathrm{Cos}[x]+4\,\mathrm{Sin}[x]}\,dx$$

Optimal (type 3, 12 leaves, 1 step):

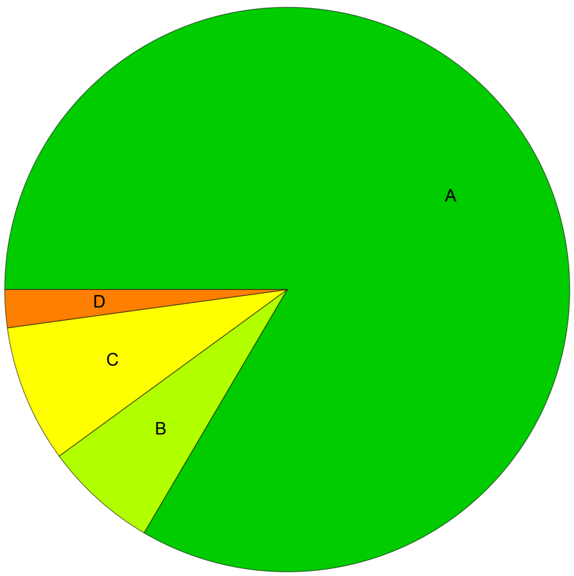
$$-\frac{1}{2+\mathrm{Tan}\left[\frac{x}{2}\right]}$$

Result (type 3, 26 leaves):

$$\frac{\mathrm{Sin}\left[\frac{x}{2}\right]}{4\,\mathrm{Cos}\left[\frac{x}{2}\right]+2\,\mathrm{Sin}\left[\frac{x}{2}\right]}$$

Summary of Integration Test Results

1892 integration problems



A - 1579 optimal antiderivatives

B - 123 more than twice size of optimal antiderivatives

C - 149 unnecessarily complex antiderivatives

D - 41 unable to integrate problems

E - 0 integration timeouts